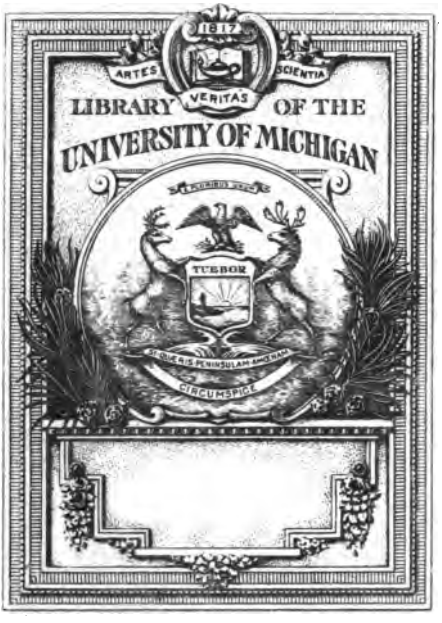
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GREENLEAF'S INTRODUCTION,
Improved Stereotype Edition.

INTRODUCTION

TO THE

NATIONAL ARITHMETIC,

ON THE

INDUCTIVE SYSTEM;

COMBINING THE

ANALYTIC AND SYNTHETIC METHODS

WITH THE

CANCELLING SYSTEM;

IN WHICH

THE PRINCIPLES OF ARITHMETIC ARE EXPLAINED AND ILLUSTRATED IN A FAMILIAR MANNER.

DESIGNED FOR COMMON SCHOOLS.

By BENJAMIN GREENLEAF, A. M.,
PRINCIPAL OF BRADFORD TEACHERS' SEMINARY.

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1845.

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☞ Other testimonials to the merits of this work, will be found in the advertising sheet, at the end of the volume, to which the reader is referred.

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PREFACE.

THE following treatise is intended for that class of pupils, who may not have sufficient time to read the larger work on this science, published by the author a few years since, and which has had extensive circulation.

It is believed, that this book contains all, that is necessary to prepare the young for the common avocations of life.

If the student wishes to obtain an extensive and full knowledge of this science, he can consult the National Arithmetic.

It has been a great object with the author to render the work *practical;* how far he has succeeded, the public must judge.

The questions are original, although there may be a similarity between some of these and others, which are before the public, and which could not be well avoided.

Although the author has carefully examined every question, yet, it is possible, some few mistakes may be found in this work. These, however, will be corrected in a future edition.

With these few prefatory remarks, the author commends this small volume to the candor of an enlightened Public.

THE AUTHOR.

BRADFORD SEMINARY,
Nov. 1st, 1842.

NOTICE.

A KEY to this work, in which the method of solving the questions is fully exhibited, has just been published, for the convenience of teachers *only.*

October 25th, 1844.

ADVERTISEMENT

TO THE

SECOND (STEREOTYPE) EDITION.

The first edition of this work having been favorably received by the public, the author is now induced carefully to revise it, and make a few additions. It is believed, that, in the present edition, all the answers to the questions will be found correct.

Great pains have been taken to make the rules and demonstrations intelligible.

In revising his work, the author has availed himself of the aid and suggestions of many practical teachers; among whom he would particularly acknowledge his obligations to two distinguished teachers in Newburyport, David P. Page, Esq., of the English High School, and Mr. Joseph Williams, of the Grammar School.

BENJAMIN GREENLEAF.

BRADFORD SEMINARY,
July 1st, 1843.

ADVERTISEMENT

TO THE

NINTH STEREOTYPE EDITION, IMPROVED.

This edition has been very carefully revised, and the author has added a Method of Cancelling, which is well adapted to the capacities of young students. He has also annexed a Supplement, containing many practical questions in the elementary principles, and in Interest, adapted to the rates per cent. which are legal in the Middle and Southern States, where this work is extensively used. The author believes these additions will increase the value and usefulness of the work, and give it a still wider circulation.

BENJAMIN GREENLEAF.

BRADFORD SEMINARY,
Oct. 25th, 1844.

CONTENTS.

CHARACTERS USED IN THIS WORK.

$ Contraction, for U. S., United States' currency, and is prefixed to dollars and cents.

= Sign of equality; as 12 inches = 1 foot, signifies, that 12 inches are equal to one foot.

+ Sign of addition; as $8+6=14$, signifies, that 8 added to 6 is equal to 14.

— Sign of subtraction; $8-6=2$, that is, 8 less 6 is equal to 2.

× Sign of multiplication; as $7\times6=42$, that is, 7 multiplied by 6 is equal to 42.

÷ Sign of division; as $42\div6=7$, that is, 42 divided by 6 is equal to 7.

$\frac{12}{6}$ Numbers placed in this manner imply, that the upper line is to be divided by the lower line.

: :: : Signs of proportion; thus, 2 : 4 :: 6 : 12, that is, 2 has the same ratio to 4, that 6 has to 12; and such numbers are called proportionals.

$\overline{15-5+3}=13$. Numbers placed in this manner show, that 5 is to be taken from 15, and 3 added to the remainder. The line at the top is called a vinculum, and connects all the numbers, over which it is drawn.

9^2 Implies, that 9 is to be raised to the second power; that is, multiplied by itself.

8^3 Implies, that 8 is to be multiplied into its square.

ARITHMETIC.

Section 1.

ARITHMETIC is the art of computing by numbers. Its five principal rules are Numeration, Addition, Subtraction, Multiplication, and Division.

NUMERATION.

Numeration teaches to express the value of numbers either by words or characters.

The numbers in Arithmetic are expressed by the following ten characters, or Arabic numeral figures, which the Moors introduced into Europe about nine hundred years ago; viz. 1 one, 2 two, 3 three, 4 four, 5 five, 6 six, 7 seven, 8 eight, 9 nine, 0 cipher, or nothing.

The first nine are called significant figures, as distinguished from the cipher, which is, of itself, insignificant.

Besides this value of those figures, they have also another, which depends on the place in which they stand, when connected together; as in the following table.

9 Hundreds of Millions.	8 Tens of Millions.	7 Millions.	6 Hundreds of Thousands.	5 Tens of Thousands.	4 Thousands.	3 Hundreds.	2 Tens.	1 Units.
9	8	7	6	5	4	3	2	1
	9	8	7	6	5	4	3	2
		9	8	7	6	5	4	3
			9	8	7	6	5	4
				9	8	7	6	5
					9	8	7	6
						9	8	7
							9	8
								9

Here any figure in the first place, reckoning from right to left, denotes only its simple value; but that in the second place, denotes ten times its simple value; and that in the third place a hundred times its simple value; and so on; the value of any successive place being always ten times its former value.

Thus in the number 1834, the 4 in the first place denotes only four units, or simply 4; 3 in the second place signifies three tens, or thirty; 8 in the third place signifies eighty tens or eight hundred; and the 1, in the fourth place, one thousand; so that the whole number is read thus, — one thousand eight hundred and thirty-four.

As to the cipher, 0, though it signify nothing of itself, yet, being joined to the right hand of other figures, it increases their value in a tenfold proportion; thus 5 signifies only five, but 50 denotes 5 tens or fifty; 500 is five hundred; and so on.

Note. — The idea of number is the latest and most difficult to form. Before the mind can arrive at such an abstract conception, it must be familiar with that process of classification, by which we successively remount from individuals to species, from species to genera, from genera to orders. The savage is lost in his attempts at enumeration, and significantly expresses his inability to proceed, by holding up his expanded fingers, or pointing to the hair of his head. See Lacroix.

ENGLISH NUMERATION TABLE.

123,	Thousands.
456;	Tridecillions.
789,	Thousands.
123;	Duodecillions.
456,	Thousands.
123;	Undecillions.
456,	Thousands.
123;	Decillions.
123,	Thousands.
456;	Nonillions.
789,	Thousands.
789;	Octillions.
323,	Thousands.
456;	Septillions.
789,	Thousands.
712;	Sextillions.
333,	Thousands.
345;	Quintillions.
789,	Thousands.
123;	Quatrillions.
137,	Thousands.
980;	Trillions.
711,	Thousands.
716;	Billions.
371,	Thousands.
712;	Millions.
456,	Thousands.
711.	Units.

To enumerate any number of figures, they must be separated by semicolons into divisions of six figures each, and each division by a comma, as in the annexed table. Each division will be known by a different name. The first three figures in each division will be so many thousands of that name, and the next three will be so many of that name, that is over its unit's place. The value of the numbers in the annexed table is, One hundred twenty-three thousand, four hundred fifty-six tridecillions; seven hundred eighty-nine thousand, one hundred twenty-three duodecillions; four hundred fifty-six thousand, one hundred twenty-three undecillions; four hundred fifty-six thousand, one hundred twenty-three decillions; one hundred twenty-three thousand, four hundred fifty-six nonillions; seven hundred eighty-nine thousand, seven hundred eighty-nine octillions; three hundred twenty-three thousand, four hundred fifty-six septillions; seven hundred eighty-nine thousand, seven hundred twelve sextillions; three hundred thirty-three thousand, three hundred forty-five quintillions; seven hundred eighty-nine thousand, one hundred twenty-three quatrillions; one hundred thirty-seven thousand, eight hundred ninety trillions; seven hundred eleven thousand, seven hundred sixteen billions; three hundred seventy-one thousand, seven hundred twelve millions; four hundred fifty-six thousand, seven hundred eleven.

NOTE. — The student must be familiar with the names from Units to Tridecillions, and from Tridecillions to Units, so that he may repeat them with facility either way.

FRENCH NUMERATION TABLE.

376,	769,	385,	123,	369,	573,	771,	127,	894,	237,	867,	123,	678,	473,	638.
Tridecillions.														
	Duodecillions.													
		Undecillions.												
			Decillions.											
				Nonillions.										
					Octillions.									
						Septillions.								
							Sextillions.							
								Quintillions.						
									Quatrillions.					
										Trillions.				
											Billions.			
												Millions.		
													Thousands.	
														Units.

It will be seen by the annexed table, that every three figures have a different name. Their value would be thus expressed, Eight hundred seventy-six tridecillions, seven hundred eighty-nine duodecillions, eight hundred thirty-five undecillions, one hundred twenty-three decillions, three hundred sixty-nine nonillions, eight hundred seventy-three octillions, seven hundred seventy-seven septillions, one hundred twenty-seven sextillions, eight hundred ninety-four quintillions, two hundred thirty-seven quatrillions, eight hundred sixty-seven trillions, one hundred twenty-three billions, six hundred seventy-eight millions, four hundred seventy-eight thousands, six hundred thirty-eight.

The pupil should write the following numbers in words.

376
611,711
3,131,671
637,313,789
63,113,716,716
143,776,711,333
44,771,631,147,671
3,761,716,137,716,716
871,137,637,471,378,637
3,761,716,137,716,167,138
611,167,637,896,431,617,761,617
671,386,131,176,378,171,714,563,813
137,471,716,756,378,817,371,767,386,389,716,473

Note. — Although the French method of enumeration is generally used, yet it may be well for the pupil to understand both the English and the French.

Section 2.

ADDITION.

MENTAL EXERCISES.

1. John had two cents and Samuel gave him two more, how many has he ?
2. Thomas had three nuts and James gave him three more, how many has he ?
3. A boy had four apples, and he found two more, how many in all ?
4. I have six dollars, and a man has paid me three more, how many have I ?
5. Enoch had seven marbles, and John gave him two more ; how many has he ?
6. Benjamin has four dollars, and his sister has three ; how many have both ?
7. Paid five dollars for a barrel of flour, and seven dollars for sugar ; how much for both ?
8. James had two cents and Samuel gave him six more ; how many has he ?
9. How many are five apples and six apples ?
10. How many are four dollars and eight dollars ?
11. How many are 2 and 3 ? 2 and 5 ? 2 and 7 ? 2 and 9 ?
12. How many are 3 and 3 ? 3 and 5 ? 3 and 7 ? 3 and 9 ?
13. How many are 4 and 3 ? 4 and 5 ? 4 and 8 ? 4 and 9 ?
14. How many are 5 and 3 ? 5 and 4 ? 5 and 7 ? 5 and 8 ? 5 and 9 ?
15. How many are 6 and 2 ? 6 and 4 ? 6 and 3 ? 6 and 5 ? 6 and 7 ? 6 and 9 ?
16. How many are 7 and 3 ? 7 and 5 ? 7 and 7 ? 7 and 6 ? 7 and 8 ? 7 and 9 ?
17. How many are 8 and 2 ? 8 and 4 ? 8 and 5 ? 8 and 7 ? 8 and 9 ? 8 and 8 ?
18. How many are 9 and 1 ? 9 and 3 ? 9 and 5 ? 9 and 4 ? 9 and 6 ? 9 and 8 ? 9 and 9 ?
19. How many are 11 and 3 ? 11 and 2 ? 11 and 4 ? 11 and 6 ? 11 and 7 ? 11 and 9 ? 11 and 11 ? 11 and

13? 11 and 12? 11 and 2 and 3? 11 and 4 and 4? 11 and 15? 12 and 7 and 3? 12 and 6 and 3? 8 and 8 and 4? 9 and 5 and 6?

20. Gave nine cents for a pound of cheese, and seven cents for a quart of molasses; what did I give for both?

21. If you buy a picture-book for eleven cents, and a knife for nine cents; what is the cost of both?

22. John paid Luke seven cents for marbles and twelve cents for gingerbread; how much money was received?

23. Thomas paid twelve cents for a top and eight cents for cherries; what did both cost?

24. A merchant sold three barrels of flour to one man and thirteen to another; what was the quantity sold?

25. I have two appletrees, one bears twelve bushels of apples, and the other eleven; how many bushels do both trees produce?

26. How many are 4 and 2 and 3? 5 and 7 and 1? 3 and 4 and 3? 6 and 6 and 5? 2 and 2 and 8? 2 and 3 and 9?

27. How many are 2 and 6 and 7? 2 and 7 and 7? 2 and 8 and 9? 2 and 7 and 4? 2 and 5 and 9? 2 and 9 and 6? 2 and 3 and 10?

28. How many are 3 and 2 and 2? 3 and 3 and 2? 3 and 5 and 5? 3 and 4 and 7? 3 and 6 and 7? 3 and 7 and 10? 3 and 8 and 9? 3 and 9 and 9?

29. How many are 4 and 2 and 2? 4 and 3 and 3? 4 and 4 and 5? 4 and 6 and 7? 4 and 7 and 7? 4 and 8 and 3? 4 and 9 and 3? 4 and 8 and 8?

30. How many are 5 and 3 and 3? 5 and 4 and 4? 5 and 5 and 1? 5 and 6 and 7? 5 and 7 and 8? 5 and 8 and 7? 5 and 9 and 9? 5 and 10 and 3?

31. How many are 6 and 2 and 7? 6 and 3 and 6? 6 and 5 and 4? 6 and 7 and 5? 6 and 8 and 7? 6 and 9 and 8? 6 and 10 and 10?

32. How many are 7 and 2 and 3? 7 and 3 and 3? 7 and 5 and 9? 7 and 6 and 6? 7 and 8 and 8? 7 and 9 and 8? 7 and 10 and 11?

33. How many are 8 and 2 and 9? 8 and 4 and 3? 8 and 7 and 7? 8 and 9 and 10? 8 and 7 and 9? 8 and 10 and 10? 8 and 9 and 12?

34. How many are 9 and 5 and 2? 9 and 4 and 3? 9 and 9 and 6? 9 and 10 and 3? 9 and 8 and 8? 9 and 4 and 9? 9 and 9 and 9?

35. How many are 2 and 2 and 4 and 5 ? 3 and 4 and 5 and 6 ? 4 and 5 and 6 and 7 ? 5 and 5 and 4 and 4 ? 9 and 1 and 2 and 3 and 5 ?
36. James had 4 apples, and Samuel gives him 5, and John gives him 6 ; how many has he ?
37. Gave 7 dollars for a barrel of flour, 5 dollars for a hundred weight of sugar, and 8 dollars for a tub of butter ; what did I give for the whole ?
38. Paid 5 dollars for a pair of boots, 12 dollars for a coat, and 6 dollars for a vest ; what was the whole cost ?
39. I have 7 appletrees, 9 cherrytrees, 6 peartrees, and 8 plumtrees ; how many in all ?
40. In a certain school, 10 scholars study grammar, 12 arithmetic, 7 logic, 2 rhetoric, and 17 punctuation ; how many are there in the school ?
41. Gave 12 cents for an almanac, 14 cents for paper, 5 cents for quills, and 8 cents for an inkstand ; what did I give for the whole ?
42. Paid 50 dollars for a horse, and 70 dollars for a chaise ; what was the price of both ?
43. A man performed a journey in 4 days ; the first day he travelled 10 miles ; the second day 12 miles ; the third day 12 miles ; the fourth day 20 miles ; what was the whole distance ?
44. Paid 2 dollars for a cap, 3 dollars for shoes, 7 dollars for pantaloons, 6 dollars for a vest, and 12 dollars for a coat ; what was the cost of the whole ?
45. Gave 75 cents for an arithmetic, and 25 cents for a geography ; what was the price of both ?
46. On the fourth of July, 20 cents were given to Emily, 15 cents to Betsey, 10 cents to Benjamin, and none to Lydia ; what did they all receive ?
47. Bought four loads of hay ; the first cost 15 dollars, the second 12 dollars, the third 20 dollars, and the fourth 17 dollars ; what was the price of the whole ?

The pupil, having performed the foregoing questions, will perceive, that

ADDITION is the collecting of numbers together to find their sum.

FOR THE SLATE.

1. I have three lots of wild land; the first contains 246 acres, the second 764 acres, and the third 918 acres; how many acres are there in the three lots?

OPERATION.

Acres.
246
764
918
1928 Ans.

In this example, the units are first added, and their sum is found to be 18; in 18 units, there are 1 *ten* and 8 units; the 8 is written under the column of units, and the 1 (ten) is carried to be added with the tens, which are found to be = 1 hundred and 2 tens; the 2 is written under the tens, and the 1 (hundred) is carried to the hundreds, which amount to 19 = 1 thousand 9 hundred; the whole of which is set down. Hence the propriety of the following

RULE.

Write units under units, tens under tens, &c. Then add upwards the units, and if the amount be less than ten, set it down. If the amount be ten or more, write down the unit figure, and carry the tens to be added with the columns of tens. Proceed in this way, till the whole is finished, writing down the total amount in the last column.

PROOF.

Begin at the top, and add together all the columns of numbers *downwards*, in the same manner as they were before added *upwards*; then if the two sums agree, the work is right.

QUESTIONS FOR THE SLATE.

2.	3.	4.	5.	6.	7.
11	47	127	678	789	1769
23	87	396	971	478	7895
97	58	787	147	719	7563
86	83	456	716	937	8765
217	275	1766	2512		

8.	9.	10.	11.	12.
876	789	123	471	1234
376	567	478	617	3456
715	743	716	871	6544
678	435	478	317	7891
910	678	127	899	8766
3555	3212	1922		

13.	14.	15.	16.
78956	71678	71123	98765
37667	12345	45678	12345
12345	67890	34680	67111
67890	34567	56777	33333
78999	89012	67812	71345
13579	78917	71444	99999
289436	354409		

17.	18.	19.
17875897	789567	37
7167512	7613	1378956
876567	761	700714
98765	123123	367
7896	70071	76117
789	475	4611779
78	1069	9171
7	374176	131765

20.	21.
8956763256 78	2345678912 34
1234567890 12	6789012345 67
8765432109 88	3210987654 33
7890123456 78	4567890123 45
2109876543 22	5432109876 55
7890123456 79	7890123456 78
4567890123 45	2109876543 22
5432109876 55	7890123456 78
3456789012 34	2109876543 22
6543210987 66	3456789012 34
1043236743 22	6543210987 66
2109876543 21	7654321087 65

22. What is the sum of the following numbers, 183, 765, 838, 375, 857, and 431 ? Ans. 3449.

23. Add the following numbers, 3791, 83, 71678, 96, 786, 4711, and 99. Ans. 81244.

24. Gave 73 dollars for a watch, 15 dollars for a cane, 119 dollars for a horse, 376 dollars for a carriage, and 7689 dollars for a house. How much did they all cost ? Ans. 8272 dollars.

25. In an orchard, 15 trees bear plums, 73 trees bear apples, 29 trees bear pears, and 14 trees bear cherries ; how many trees are there in the orchard ? Ans. 131 trees.

26. The hind quarters of an ox weighed 375 pounds each ; the fore quarters 315 pounds each ; the hide weighed 96 pounds, and the tallow 87 pounds. What was the whole weight of the ox ? Ans. 1563 pounds.

27. A man bought a farm for 1728 dollars, and sold it so as to gain 375 dollars ; how much did he sell it for ? Ans. 2103 dollars.

28. A merchant bought five pieces of cloth. For the first he gave 376 dollars ; for the second 198 dollars ; for the third 896 dollars ; for the fourth 691 dollars ; for the fifth 96 dollars. How much did he give for the whole ? Ans. 2257 dollars.

29. A merchant bought five hogsheads of molasses for 375 dollars, and sold it so as to gain 25 dollars on each hogshead ; for how much did he sell it ? Ans. 500 dollars.

30. John Smith's farm is worth 7896 dollars ; he has bank stock valued at 369 dollars ; and he has in cash 850 dollars. What is he worth ? Ans. 9115 dollars.

31. Required the number of inhabitants in the New England States, there being in Maine 501,793 ; in New Hampshire 284,574 ; in Massachusetts 737,699 ; in Rhode Island 108,830 ; in Connecticut 309,978 ; in Vermont 291,948. Ans. 2,234,822.

32. Required the number of inhabitants in the Middle States, there being in New York 2,428,921 ; in New Jersey 373,306 ; in Pennsylvania 1,724,033 ; in Delaware 78,085 ; in Maryland 469,232. Ans. 5,073,577.

33. Required the number of persons in the Southern States, there being in Virginia 1,239,797 ; in North Carolina 753,419 ; in South Carolina 594,398 ; in Georgia

691,392; in Alabama 590,756; in Mississippi 375,651; in Louisiana 352,411. Ans. 4,597,824.

34. How many inhabitants in the Western States, there being in Tennessee 829,210; in Kentucky 779,828; in Ohio 1,519,467; in Indiana 685,866; in Illinois 476,183; in Missouri 383,702; in Arkansas 97,574; in Michigan 212,267? Ans. 4,984,097.

35. How many inhabitants in the following Territories and the District of Columbia, there being in Florida 54,477; in Wisconsin 30,945; in Iowa 43,112; and in the District of Columbia 43,712? Ans. 172,246.

36. How many are the inhabitants of the United States, there being in New England 2,234,822; in the Middle States 5,073,577; in the Southern States 4,597,824; in the Western States 4,984,097; in the Territories 172,246? Ans. 17,062,566.

Section 3.

SUBTRACTION.

MENTAL OPERATIONS.

1. James has three dollars, and John has two dollars; how many has James more than John?
2. Thomas had five oranges, he gives two to John; how many has he left?
3. Peter had six marbles, he gives two to Samuel; how many has he left?
4. Lydia had four cakes, having lost one; how many has she left?
5. Daniel having eight cents, he gives three to Mary; how many has he left?
6. Benjamin had ten nuts, he gives four to Jane, and three to Emily; how many has he left?
7. Moses gives eleven oranges to John, and eight to Enoch; how many more has John than Enoch?
8. Agreed to labor for a man twelve days? how many remain, after I have been with him five days?

9. I owed Thomas nine dollars, and having paid him seven ; how many remain due ?

10. From ten dollars, I paid four dollars and three dollars ; how much have I left ?

11. Timothy had eleven marbles, he lost seven ; how many had he left ?

12. John is thirteen years old, and his brother Thomas is seven ; how much older is John than Thomas ?

13. From 15 dollars, I paid five ; how many have I left ?

14. Sold a barrel of flour for eight dollars, and a bushel of wheat for two dollars ; what was the difference in the prices ?

15. Paid seven dollars for a pair of boots, and two dollars for shoes ; how much did the boots cost more than the shoes ?

16. How many are 4 less 2 ? 4 less 1 ? 4 less 4 ?

17. How many are 4 less 3 ? 5 less 1 ? 5 less 5 ?

18. How many are 5 less 2 ? 5 less 3 ?

19. How many are 6 less 1 ? 6 less 2 ? 6 less 4 ? 6 less 5 ?

20. How many are 7 less 2 ? 7 less 3 ? 7 less 4 ? 7 less 6 ?

21. How many are 8 less 6 ? 8 less 5 ? 8 less 2 ? 8 less 4 ? 8 less 1 ?

22. How many are 9 less 2 ? 9 less 4 ? 9 less 5 ? 9 less 7 ? 9 less 3 ?

23. How many are 10 less 8 ? 10 less 7 ? 10 less 5 ? 10 less 3 ? 10 less 1 ?

24. How many are 11 less 9 ? 11 less 7 ? 11 less 5 ? 11 less 3 ? 11 less 4 ?

25. How many are 12 less 10 ? 12 less 8 ? 12 less 6 ? 12 less 4 ? 12 less 7 ?

26. How many are 13 less 11 ? 13 less 10 ? 13 less 7 ? 13 less 9 ? 13 less 5 ?

27. How many are 14 less 11 ? 14 less 9 ? 14 less 8 ? 14 less 6 ? 14 less 7 ? 14 less 3 ?

28. How many are 15 less 2 ? 15 less 4 ? 15 less 5 ? 15 less 7 ? 15 less 9 ? 15 less 13 ?

29. How many are 16 less 3 ? 16 less 4 ? 16 less 7 ? 16 less 9 ? 16 less 11 ? 16 less 15 ?

30. How many are 17 less 1? 17 less 3? 17 less 5? 17 less 7? 17 less 8? 17 less 12?
31. How many are 18 less 2? 18 less 4? 18 less 7? 18 less 8? 18 less 10? 18 less 12?
32. How many are 19 less 1? 19 less 3? 19 less 5? 19 less 7? 19 less 9? 19 less 16?
33. How many are 20 less 5? 20 less 8? 20 less 9? 20 less 12? 20 less 15? 20 less 19?
34. How many are 30 less 5? 30 less 10? 30 less 15? 30 less 20? 30 less 25?
35. Bought a horse for 63 dollars, and sold him for 70; what did I gain?
36. Sold a barrel of flour for 8 dollars, which cost me 10 dollars; what did I lose?
37. John travels 25 miles a day, and Samuel 32 miles? what is the difference?
38. I have 100 dollars, and after I shall have given 17 to Benjamin, and paid a debt of 30 dollars to J. Smith; how many dollars have I left?

The pupil, having performed the above, will perceive, that

SUBTRACTION teaches to take a less number from a greater, and to find the difference.

FOR THE SLATE.

1. If I have 624 dollars and lose 342 of them, how many remain?

OPERATION.

From	624
Take	342
	282

In this question, we take the 2 units from 4 units and 2 units remain, which we write down under units, as the first figure in the answer. In attempting to take the 4 tens, we find a difficulty, as 4 cannot be taken from 2. We therefore borrow 1 (hundred) from the 6 (hundred), which being equal to 10 tens, we add it to the 2 tens in the upper line, making 12 tens, and 8 (tens) remain, which we set down. We then proceed to the hundreds. As we have borrowed 1 from the 6 hundreds, the 6 is too large by 1. We must, therefore, take the 3 from 5, and we find 2 (hundreds) remain, which we set down.

Or because the 6 is too large by 1, we may add 1 to the 3 and say 4 from 6 = 2. This process is called borrowing and carrying. Hence the following

RULE.

Place the less number under the greater; units under units, tens under tens, &c. Begin with the units; and, if the lower figure be smaller than the upper, take it therefrom, and write the difference below; but, if the upper figure be less than the lower figure, add ten to the upper one, and place the difference between them under the units as before, and carry one to the next number at the bottom, and proceed thus, till all the numbers are subtracted.

Note. The upper line is called the Minuend, and the lower one the Subtrahend. The result of the question is called the Remainder.

PROOF.

Add the Remainder to the Subtrahend, and, if their sum be like the Minuend, the work is right.

QUESTIONS FOR THE SLATE.

	2. £	3. Cwt.	4. Miles.	5. Bushels.
Minuend,	789	376	531	4789050
Subtrahend,	346	187	389	1789582
	443	189	142	2999468

	6. Tons.	7. Gallons.	8. Pecks.	9. Feet.
From	978	67158	14711	100000
Take	199	14339	9197	90909
	779	52819		

	10. Miles.	11. Dollars.	12. Minutes.	13. Seconds.
From	67895	456798	765321	555555
Take	19999	190899	177777	177777

	14. Rods.	15. Acres.
From	1002003004005 00	1000000000000
Take	908070605040 30	999999999999

16. From 1728 dollars, I paid 961 dollars; how many remain? Ans. 767 dollars.

17. Independence was declared in 1776; how many years from this period to the close of the last war, in 1815? Ans. 39 years.

18. The last transit of Venus was 1769, and the next will be 1874, how many years will intervene?
Ans. 105 years.

19. In 1830, the number of inhabitants in Bradford was 1856; and in 1840 it was 2222; what was the increase? Ans. 366.

20. How many more inhabitants are there in New York city than in Boston, there being, by the last census, 312,710 inhabitants in the former, and 93,383 in the latter city? Ans. 219,327 inhabitants.

21. In 1821 there were imported into the United States 21,273,659 pounds of coffee, and in 1839, 106,696,992 pounds; what was the increase?
Ans. 85,423,333 pounds.

22. By the last census, 11,853,507 bushels of wheat are raised in New York, and 13,029,756 bushels in Pennsylvania; how many bushels in the latter State more than the former? Ans. 1,176,249 bushels.

23. The real estate of James Dow is valued at 3,769 dollars, and his personal estate at 2,648 dollars; he owes John Smith 1,728 dollars, and Job Tyler 1,161 dollars; how much is J. Dow worth? Ans. 3528 dollars.

24. If a man receive 5 dollars per day for labor, and it cost him 2 dollars per day to support his family; what will he have accumulated at the close of one week? Ans. 18 dollars.

25. The city of New York owes 9,663,269 dollars, and Boston owes 1,698,232 dollars; how much more does New York owe than Boston?
Ans. 7,965,037 dollars.

26. From five hundred eighty-one thousand take three thousand and ninety-six. Ans. 577,904.

27. E. Webster owns 6,765 acres of land, and he gave to his oldest brother 2,196 acres, and his uncle Rollins 1,981 acres; how much has he left?
Ans. 2,588 acres.

Section 4.

MULTIPLICATION.

TABLE OF PYTHAGORAS.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84	88	92	96
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120	126	132	138	144
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140	147	154	161	168
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160	168	176	184	192
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180	189	198	207	216
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220	231	242	253	264
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264	276	288
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260	273	286	299	312
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280	294	308	322	336
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320	336	352	368	384
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340	357	374	391	408
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360	378	396	414	432
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380	399	418	437	456
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480
21	42	63	84	105	126	147	168	189	210	231	252	273	294	315	336	357	378	399	420	441	462	483	504
22	44	66	88	110	132	154	176	198	220	242	264	286	308	330	352	374	396	418	440	462	484	506	528
23	46	69	92	115	138	161	184	207	230	253	276	299	322	345	368	391	414	437	460	483	506	529	552
24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	384	408	432	456	480	504	528	552	576

MENTAL OPERATIONS.

1. What cost three bushels of wheat at three dollars per bushel ?
2. What cost 5 barrels of flour at 6 dollars per barrel ?
3. What cost 6 bushels of beans at 2 dollars per bushel ?
4. What cost 5 quarts of cherries at 7 cents per quart ?
5. What will 7 gallons of vinegar cost at 12 cents per quart ?
6. What cost 9 acres of land at 10 dollars per acre ?
7. If a pint of currants cost 4 cents, what cost 9 quarts ?
8. If, in 1 penny, there are 4 farthings, how many in 9 pence ? In 7 pence ? In 8 pence ? In 4 pence ? In 3 pence ?
9. If 12 pence make a shilling, how many pence in 3 shillings ? In 5 shillings ? In 7 shillings ? In 9 shillings ?
10. If 4 pecks make a bushel, how many pecks in 2 bushels ? In 3 bushels ? In 4 bushels ? In 6 bushels ? In 7 bushels ? In 9 bushels ?
11. If 12 inches make 1 foot, how many inches in 3 feet ? In 4 feet ? In 5 feet ? In 7 feet ? In 8 feet ? In 9 feet ? In 10 feet ? In 12 feet ?
12. If there be 9 feet in a square yard, how many feet in 4 yards ? In 5 yards ? In 6 yards ? In 8 yards ? In 9 yards ? In 12 yards ?
13. What cost 3 yards of cloth at 5 dollars per yard ? 4 yards ? 5 yards ? 6 yards ? 7 yards ? 8 yards ? 9 yards ? 10 yards ? 11 yards ? 12 yards ? 20 yards ?
14. If 1 pound of iron cost 7 cents, what cost 2 pounds ? 3 pounds ? 5 pounds ? 6 pounds ? 7 pounds ? 8 pounds ? 9 pounds ? 12 pounds ?
15. If 1 pound of raisins cost 6 cents, what cost 4 pounds ? 5 pounds ? 6 pounds ? 7 pounds ? 8 pounds ? 9 pounds ? 10 pounds ? 12 pounds ?
16. In 1 acre there are 4 roods, how many roods in 2 acres ? In 3 acres ? In 4 acres ? In 5 acres ? In 6 acres ? In 9 acres ?
17. A good pair of boots is worth 5 dollars ; what must I give for 5 pair ? For 6 pair ? For 7 pair ? For 8 pair ?
18. A cord of good walnut wood may be obtained for 8 dollars ; what must I give for 4 cords ? For 6 cords ? For 9 cords ?

19. A gallon of molasses is worth 25 cents, what is the value of 2 gallons? Of 3 gallons? Of 4 gallons? Of 5 gallons? Of 6 gallons?

20. What cost 4 quarts of milk at 5 cents a quart? and 8 gallons of vinegar at 10 cents a gallon?

21. If a man earn 7 dollars a week, how much will he earn in 3 weeks? In 4 weeks? In 5 weeks? In 6 weeks? In 7 weeks? In 9 weeks?

22. If one thousand feet of boards cost 12 dollars, what cost 4 thousand? 5 thousand? 6 thousand? 7 thousand? 12 thousand?

23. In 1 pound there are 20 shillings, how many shillings in 3 pounds? In 4 pounds? In 6 pounds? In 9 pounds?

24. If 3 pair of shoes buy 1 pair of boots, how many pair of shoes will it take to buy 7 pair of boots?

25. If 5 bushels of apples buy 1 barrel of flour, how many bushels of apples are equal in value to 12 barrels of flour?

The foregoing questions having been performed, it will be perceived, that

MULTIPLICATION is a compendious way of performing Addition, and that it consists of three parts; the multiplicand, or number to be multiplied; the multiplier, or number to multiply by; and the result, which is called the product.

The pupil, having thoroughly committed the multiplication Table, will notice the following

RULE.

Place the larger number uppermost, and then set the multiplier under it, so that units may be under units, &c., and multiply by the multiplier, beginning at the unit's place and carry for tens as in addition.

When the multiplier consists of more places than one, multiply each figure in the multiplicand by every figure in the multiplier, beginning with the units, and placing the first figure of each product directly under its multiplier, then add all their several products together in the same order, as they stand, and their sum will be the true product required.

When there are ciphers between the significant figures of the multipliers, omit them, and multiply by the significant figures only.

If there be ciphers at the right hand of the multiplier or multiplicand, they may be neglected in the operation, but their number must be affixed to the product.

PROOF.

Multiplication may be proved by division, or by multiplying the multiplier by the multiplicand, as in 12th and 13th questions, or by casting out the 9's, thus; cast the 9's from the multiplicand and place the remainder at the *right* hand of a cross, then cast the 9's from the multiplier and set the remainder at the *left* hand of the cross, then cast the 9's from the product, and set the remainder at the *top* of the cross. Multiply the numbers together on each side of the cross, and cast the 9's from their product, and if the remainder be like the number at the *top* of the cross, it may be presumed the work is right. See question 14.

QUESTIONS FOR THE SLATE.

	1.	2.	3.
Multiplicand	8756	4567	7896
Multiplier	4	3	5
	35024	13701	

4.	5.	6.	7.
56807	47893	71657	89765
5	6	7	9
284035	287358		

8.	9.	10.	11.
67895	78956	89325	47896
36	47	91	82
407370	552692		
203685	315824		
2444220	3710932		

12.	13.	14.	
7895	3456	12345	3
3456	7895	2231	8×6
47370	17280	12345	3
39475	31104	37035	
31580	27648	24690	
23685	24192	24690	
27285120	27285120	27541695	

15.	16.
878532400	713378900
3200	70080
175706480000	57070312000
26355972	49936523
2811303680000	49993593312000

	Answers.
17. Multiply 767853 by 9.	6910677.
18. Multiply 876538765 by 8.	7012310120.
19. Multiply 7654328 by 7.	53580296.
20. Multiply 4976387 by 5.	24881935.
21. Multiply 8765448 by 12.	105185376.
22. Multiply 4567839 by 11.	50246229.
23. Multiply 68759 by 5678.	390413602.
24. Multiply 78113 by 70005.	5468300565.
25. Multiply 46700 by 60103.	2806810100.
26. Multiply 83000 by 10007.	830581000.
27. Multiply 40009 by 40009.	1600720081.

28. What cost 14 barrels of apples at 3 dollars per barrel? Ans. 42 dollars.

29. What cost 17 tons of hay at 18 dollars per ton? Ans. 306 dollars.

30. What cost 47 cords of wood at 7 dollars per cord? Ans. 329 dollars.

31. What cost 47 hogsheads of molasses at 13 dollars per hogshead? Ans. 611 dollars.

32. What cost 97 oxen at 29 dollars each? Ans. 2813 dollars.

33. Sold a farm containing 367 acres, what was the amount at 97 dollars per acre? Ans. 35599 dollars.

34. An army of 17006 men receive each 109 dollars as their annual pay; what is the amount paid the whole army? Ans. 1853654 dollars.

35. If a mechanic deposit annually in the Savings Bank, 407 dollars, what will be the sum deposited in 27 years? Ans. 10989 dollars.

36. If a man travel 37 miles in one day, how far will he travel in 365 days? Ans. 13505 miles.

37. If there be 24 hours in one day, how many hours in 365 days? Ans. 8760 hours.

38. How many gallons are in 87 hogsheads, there being 63 gallons in each hogshead? Ans. 5481 gallons.

39. If the expenses of the Massachusetts Legislature be 1839 dollars per day, what will be the amount in a session of 109 days? Ans. 200451 dollars.

40. If a hogshead of sugar contains 368 pounds, how many pounds in 187 hogsheads? Ans. 68816 pounds.

Section 5.

DIVISION.

MENTAL EXERCISES.

1. A gentleman divided 6 apples between 2 boys; how many did each receive?

2. A farmer received 8 dollars for 2 sheep; what was the price of each?

3. A man gave 15 dollars for 3 barrels of flour; what was the cost of each barrel?

4. A lady divided 20 oranges among her 5 daughters; how many did each receive?

5. If 4 casks of lime cost 12 dollars, what is the value of 1 barrel?

6. A laborer earned 48 shillings in 6 days; what did he receive per day?

7. A man can perform a certain piece of labor in 30 days; how long will it take 5 men to do the same?

8. When 72 dollars are paid for 8 acres of land; what cost 1 acre? What cost 3 acres?
9. If 21 pounds of flour can be obtained for 3 dollars, how much can be obtained for 1 dollar? How much for 8 dollars? How much for 9 dollars?
10. Gave 56 cents for 8 pounds of raisins; what cost 1 pound? What 7 pounds?
11. If a man walk 24 miles in 6 hours, how far will he walk in 1 hour? How far in 10 hours?
12. Paid 56 dollars for 7 hundred weight of sugar; what cost 1 hundred weight? What cost 10 hundred weight?
13. If 5 horses will eat a load of hay in one week, how long would it last one horse?
14. In 20, how many times 2? How many times 4? How many times 5? How many times 10?
15. In 24 how many times 3? How many times 4? How many times 6? How many times 8?
16. How many times 7 in 21? In 28? In 56? In 35? In 14? In 63? In 77? In 70? In 84?
17. How many times 6 in 12? In 36? In 18? In 54? In 60? In 42? In 48? In 72? In 66?

The pupil will now perceive, that

DIVISION is a short or compendious way of performing Subtraction.

Its object is to find how often one number is contained in another. It consists of four parts, the dividend, or number to be divided; the divisor, the number we divide by; the quotient, which shows how many times the divisor is contained in the dividend; and the remainder, which is always less than the divisor, and of the same name of the dividend.

I. When the divisor is less than 13, the question should be performed by

SHORT DIVISION.

1. Divide 7554 dollars equally among 6 men.

Dividend.
Divisor 6) 7 5 5 4
Quotient 1 2 5 9

In performing this question, inquire how many times 6, the divisor, is contained in 7, which is 1 time, and 1 remaining; write the

1 under the 7, and suppose 1, the remainder, to be placed before the next figure of the dividend, 5 ; and the number would be 15. Then inquire how many times 6, the divisor, is contained in 15. It is found to be 2 times, and 3 remaining. Write the 2 under the 5, and suppose the remainder, 3, to be placed before the next figure of the dividend, 5 ; and the number would be 35. Inquire again how many times 35 will contain the divisor, 6. It is found to be 5 times, and 5 remaining. Write the 5 under the 5 in the dividend, and suppose the remainder, 5, to be placed before the last figure of the dividend, 4 ; and the number would be 54. Lastly, inquire how many times 54 will contain the divisor, 6. It is found to be 9 times, which we place under the 4 in the dividend. Thus we find, that each man will receive 1259 dollars.

From the above illustration we deduce the following

RULE.

See how many times the divisor may be contained in the first figure or figures of the dividend, and place the result immediately under that figure; and what remains suppose to be placed directly before the next figure of the dividend; and then inquire how many times these two figures will contain the divisor, and place the result as before; and so proceed until the question is finished.

2.	3.	4.
3)7893762	4)4763256	5)3789565
2631254	1190814	

5.	6.	7.
6)8765389	7)987635	8)378532

8.	9.	10.
9)8953784	11)7678903	12)6345321

Quotients.

11. Divide 479956 by 6. 79992$\frac{4}{6}$.
12. Divide 385678 by 7. 55096$\frac{6}{7}$.
13. Divide 438789 by 8. 54848$\frac{5}{8}$.
14. Divide 1678767 by 9. 186529$\frac{6}{9}$.
15. Divide 11497583 by 12. 958131$\frac{11}{12}$.

16. Divide 944,580 dollars equally among 12 men, and what will be the share of each ? Ans. 78,715 dollars.

17. Divide 154,503 acres of land equally among 9 persons. Ans. 17,167 acres.

18. A plantation in Cuba was sold for 7,011,608 dollars, and the amount was divided among 8 persons. What was paid to each person ? Ans. 876,451 dollars.

	Quotients.	Rem.
19. Divide 5678956 by 5.		1.
20. Divide 1135791 by 7.		6.
21. Divide 1622550 by 8.		6.
22. Divide 2028180 by 9.		3.
23. Divide 2253530 by 12.		2.
24. Divide 1877940 by 11.		9.
Sum of the quotients,	2084732.	27.

25. A prize, valued at 178,656 dollars, is to be equally divided among 12 men ; what is the share of each ? Ans. 14,888 dollars.

26. Among 7 men, 67,123 bushels of wheat are to be distributed ; how many bushels does each man receive ? Ans. 9,589 bushels.

27. If 9 square feet make 1 square yard, how many yards in 895,347 square feet ? Ans. 99,483 yards.

28. A township of 876,136 acres is to be divided among 8 persons ; how many acres will be the portion of each ? Ans. 109,517 acres.

29. Bought a farm for 5670 dollars, and sold it for 7896 dollars, and I divide the net gain among 6 persons ; what does each receive ? Ans. 371 dollars.

30. If 6 shillings make a dollar, how many dollars in 7890 shillings ? Ans. 1315.

II. When the divisor exceeds 12, the operation should be performed by

LONG DIVISION,

as in the following question.

31. A gentleman divided equally among his 19 sons, 4712 dollars ; what is the share of each ?

OPERATION.

Dividend.

Divisor. 19)4712(248 *Quotient.*
38 19
91 2232
76 248
152 4712 *Proof.*
152
000 *Remainder.*

The object of this question is to find how many times 4712 will contain 19, or how many times 19 must be subtracted from 4712, until nothing remains. We first inquire how many times 19 may be contained in 47 (thousand). Having found it to be 2 (hundred) times, we write 2 in the quotient and multiply it by the divisor, 19, and place their product under 47, from which we subtract it, and find the remainder to be 9, to which we annex the next figure in the dividend, 1. And having found that 91 (tens) will contain the divisor, 19, 4 (tens) times, we write 4 in the quotient, multiply it by 19, and place the product 76 under 91, from which we subtract it, and, to the remainder, 15 (tens), we annex the last figure of the dividend, 2, and inquire how many times 152 will contain 19, and we find it to be 8 times; and having placed the product of 8 times 19, that is, 152, under the 152, we find there is no remainder, and that the number 4712 will contain 19, the divisor, 248 times; that is, each man will receive 248 dollars.

To prove our operation is correct, we reason thus. If one man receive 248 dollars, 19 men will receive 19 times as much, and 19 times 248 are 4712, the same as the dividend; and this operation is effected by multiplying the divisor by the quotient, and adding in the remainder if there be one. The student will now see the propriety of the following

RULE.

Place the divisor before the dividend, and inquire how many times it is contained in a competent number of figures in the dividend, and place the result in the quotient; multiply the figure in the quotient by the divisor, and place the product under those figures in the dividend, in which it was inquired, how many times the divisor was contained; subtract this product from the dividend, and to the remainder

bring down the next figure of the dividend; and then inquire how many times this number will contain the divisor, and place the result in the quotient, and proceed as before, until all the figures of the dividend are brought down.

Note 1. — It will sometimes happen, that, after a figure is brought down, the number will not contain the divisor; a cipher is then placed in the quotient, and another figure is brought down, and so continue until it will contain the divisor, placing a cipher each time in the quotient.

Note 2. — The remainder in all cases is less than the divisor, and of the same denomination of the dividend; and, if at any time, we subtract the product of the figure in the quotient and divisor from the dividend, and the remainder is more than the divisor, the figure in the quotient is not large enough.

PROOF.

Division may be proved by Multiplication, Addition, or by Division itself.

To prove it by Multiplication, the divisor must be multiplied by the quotient, and, to the product, the remainder must be added, and, if the result be like the dividend, the work is right.

To prove it by Addition. Add up the several products of the divisor and quotient with the remainder, and, if the result be like the dividend, the work is right.

To prove it by Division itself. Subtract the remainder from the dividend, and divide this number by the quotient, and the quotient found by this division will be equal to the former divisor, when the work is right.

32.

```
83)148678(  1791
   83*
   656
   581*
    757
    747*
     108
      83*
      25*
   148678
```

```
14328
148653
   25
148678 Proof.
```

33.

```
427)567896(1329
   427
   1408
   1281
    1279
     854
     4256
     3843
      413
```

* Note. The asterisms show the numbers to be added.

34.

144)13824(96
1296
864
864

35.

96)13824(144
96
422
384
384
384

NOTE. The 34th question is proved by the 35th.

36.

86|000)8963|486(104
86
363
344
19486

37.

Rem.
1|0000)7|8967(7 *Quotient.*

		Quotients.	Remainders.
38.	Divide 867532 by 59.	14703.	55.
39.	Divide 167008 by 87.	1919.	55.
40.	Divide 345678 by 379.	912.	30
41.	Divide 6789563 by 1234.		95.
42.	Divide 78112345 by 8007.		4060.
43.	Divide 34533669 by 9999.		7122.
44.	Divide 99999999 by 3333.		0.
45.	Divide 47856712 by 1789.		962.
46.	Divide 13112297 by 8900.		2597.
47.	Divide 10000000 by 7007.		1011.
48.	Divide 15678953 by 8790.		6383.
49.	Divide 71800100 by 4701.		1727.

III. To multiply by a fraction.

RULE.

Multiply the given number by the numerator of the fraction, and divide the product by the denominator. If any thing remain place it over the divisor at the right hand of the quotient.

NOTE. When the number is such, that it may be divided by the denominator without a remainder, the better way is to divide the given number by the denominator, and multiply the quotient by the numerator. This is the analytical method.

50. What is $\frac{3}{4}$ of 144?

Synthetic method.	Analytical method.
144	4)144
3	36
4)432	3
108 Ans.	108 Ans.

Divide by 4 to get one fourth, and multiply by 3 to get 3 fourths.

51. What is $\frac{5}{8}$ of 365? Ans. $228\frac{1}{8}$.

52. What is $\frac{3}{7}$ of 128? Ans. $54\frac{6}{7}$.

53. What is $\frac{6}{11}$ of 386? Ans. $210\frac{6}{11}$.

54. Sold a farm for 1728 dollars; and, if I give $\frac{5}{12}$ of this sum to indigent persons, what do they receive?

Ans. 720 dollars.

55. If from 1000 dollars $\frac{3}{8}$ be taken, what sum will remain? Ans. 625 dollars.

IV. To divide by a fraction.

RULE.

Multiply the given number by the denominator, and divide the product by the numerator.

56. Divide 125 by $\frac{5}{8}$.

125	In this example, we multiply by 8 to reduce the 125 to eighths; and then we see how often 5 (eighths) are contained in them.
8	
5)1000	
200	

57. Sold $\frac{7}{8}$ of a house for 3227 dollars; what was the value of the whole house? Ans. 3688 dollars.

V. To divide by a composite number, that is, a number, which is produced by the multiplying of two or more numbers.

RULE.

Divide the dividend by one of these numbers, and the quotient thence arising by another, and so continue; and the last quotient will be the answer.

NOTE. To find the true remainder, we multiply the last remainder by the last divisor but one, and to the product add the next preced-

ing remainder; we multiply this product by the next preceding divisor, and to the product add the next preceding remainder; and so on until we have gone through all the divisors and remainders to the first.

58. Divide 67872 by 24.

OPERATION.

4)67872
6)16968
2828

In this question, we divide by 4 and 6, because they are the factors, or composite numbers of 24.

		Quotients.
59. Divide 765325 by	$25 = 5 \times 5$.	30613.
60. Divide 123396 by	$84 = 7 \times 12$.	1469.
61. Divide 611226 by	$81 = 9 \times 9$.	7546.
62. Divide 987625 by	$125 = 5 \times 5 \times 5$.	7901.

Section 6.

APPLICATION OF THE PRECEDING RULES.

1. A farmer bought 5 yoke of oxen at 87 dollars a yoke; 37 cows at 37 dollars each; 89 sheep at 3 dollars a piece. He sold the oxen at 98 dollars a yoke; for the cows he received 40 dollars each; and, for the sheep, he had 4 dollars a piece; what did he gain by his trade? Ans. 255 dollars.

2. In 4008 hours, how many days? Ans. 167 days.

3. In 169 weeks, how many days? Ans. 1183 days.

4. If 12 inches make a foot, how many feet in 48096 inches? Ans. 4008 feet.

5. In 15300 minutes, how many hours? Ans. 255 hours.

6. If 144 inches make 1 square foot, how many square feet in 20736 inches? Ans. 144 feet.

7. An acre contains 160 square rods; how many in a farm containing 769 acres? Ans. 123040 rods.

8. A gentleman bought a house for three thousand forty-seven dollars, and a carriage and span of horses for five hundred seven dollars. He paid at one time, two thousand seventeen dollars, and at another time, nine hundred seven dollars. How much remains due? Ans. 630 dollars.

9. The erection of a factory cost 68,255 dollars; supposing this sum to be divided into 365 shares, what is the expense of each? Ans. 187 dollars.

10. A gentleman, possessing an estate of 375,846 dollars, bequeathed 7,494 dollars to the Bible Society; 4,230 dollars for the support of schools; and one third to his wife; the remainder was to be equally divided among his 12 sons and 8 daughters; what sum will each receive? Ans. 11,942 dollars.

11. There were distilled in the United States in 1840, thirty-six millions three hundred forty-three thousand two hundred thirty-six gallons of ardent spirits; and the number of free white males, over 15 years, is four millions seventy-four thousand nine hundred fifteen; now supposing the liquor to be drank by one third of those persons in one year, what quantity would each consume? Ans. More than 26 gallons.

12. A man gave half of his estate to his wife, one third of what remained to his son, and the residue was equally divided among his 7 daughters, who received each 124 dollars; what was the whole estate? Ans. 2,604 dollars.

Section 7.

TABLES OF MONEY, WEIGHTS, AND MEASURES.

UNITED STATES' MONEY.

10 Mills	make	1 Cent,	marked	c.
10 Cents	"	1 Dime,	"	d.
10 Dimes	"	1 Dollar,	"	$.
10 Dollars	"	1 Eagle,	"	E.

Mills.		Cents.		Dimes.		Dollars.		Eagles.
10	=	1						
100	=	10	=	1				
1000	=	100	=	10	=	1		
10000	=	1000	=	100	=	10	=	1

ENGLISH MONEY.

4 Farthings	make	1 Penny,	marked	d.
12 Pence	"	1 Shilling,	"	s.
20 Shillings	"	1 Pound,	"	£.

qrs.		d.		s.		£
4	=	1				
48	=	12	=	1		
960	=	240	=	20	=	1

FRENCH MONEY.

100 Centimes make 1 Franc = .1875 dollar.

TROY WEIGHT.

24 Grains	make	1 Pennyweight,	marked	dwt.
20 Pennyweights	"	1 Ounce,	"	oz.
12 Ounces	"	1 Pound,	"	lb.

grs.		dwt.		oz.		lb.
24	=	1				
480	=	20	=	1		
5760	=	240	=	12	=	1

By this weight are weighed gold, silver, and jewels.

NOTE. "The original of all weights, used in England, was a grain or corn of wheat, gathered out of the middle of the ear; and being well dried, 32 of them were to make one pennyweight, 20 pennyweights one ounce, and 12 ounces one pound. But, in later times, it was thought sufficient to divide the same pennyweight into 24 equal parts, still called grains, being the least weight now in common use; and from hence the rest are computed."

APOTHECARIES' WEIGHT.

20 Grains	make	1 Scruple,	marked	sc. or ℈.
3 Scruples	"	1 Dram,	"	dr. or ʒ
8 Drams	"	1 Ounce,	"	oz. or ℥
12 Ounces	"	1 Pound,	"	lb. or ℔.

gr.		sc.		dr.		oz.		lb.
20	=	1						
60	=	3	=	1				
480	=	24	=	8	=	1		
5760	=	288	=	96	=	12	=	1

Apothecaries mix their medicines by this weight; but buy and sell by Avoirdupois. The pound and ounce of this weight are the same as in Troy Weight.

AVOIRDUPOIS WEIGHT.

16 Drams	make	1 Ounce,	marked	oz,
16 Ounces	"	1 Pound,	"	lb.
28 Pounds	"	1 Quarter,	"	qr.
4 Quarters	"	1 Hundred weight,	"	cwt.
20 Hundred weight	"	1 Ton,	"	ton.

dr.		oz.		lb.		qr.		cwt.		ton.
16	=	1								
256	=	16	=	1						
7168	=	448	=	28	=	1				
28672	=	1792	=	112	=	4	=	1		
573440	=	35840	=	2240	=	80	=	20	=	1

By this weight are weighed almost every kind of goods, and all metals except gold and silver. By a late law of Massachusetts, the cwt. contains 100 lbs. instead of 112 lbs.

LONG MEASURE.

12 Inches	make	1 Foot,	marked	ft.
3 Feet	"	1 Yard,	"	yd.
5½ Yards, or 16½ feet	"	1 Rod, or pole,	"	rd.
40 Rods	"	1 Furlong,	"	fur.
8 Furlongs	"	1 Mile,	"	m.
3 Miles	"	1 League,	"	lea.
69½ Miles (nearly)	"	1 Degree,	"	Deg. or °.
360 Degrees	"	1 Circle of the Earth.		

in.		ft.		yd.		rd.		fur.		m.
12	=	1								
36	=	3	=	1						
198	=	16½	=	5½	=	1				
7920	=	660	=	220	=	40	=	1		
63360	=	5280	=	1760	=	320	=	8	=	1

CLOTH MEASURE.

2¼ Inches	make	1 Nail,	marked	na.
4 Nails	"	1 Quarter of a yard	"	qr.
4 Quarters	"	1 Yard,	"	yd.
3 Quarters	"	1 Ell Flemish,	"	E. F.
5 Quarters	"	1 Ell English,	"	E. E.

NOTE. The Ell French is not in use.

SQUARE MEASURE.

144	Square inches	make	1	Square foot,	marked ft.
9	Square feet	"	1	Square yard,	" yd.
30¼	Square yards	"	1	Square rod or pole,	" p.
272¼	Square feet	"	1	Square rod or pole,	" p.
40	Square rods or poles	"	1	Rood,	" R.
4	Roods	"	1	Acre,	" A.
640	Acres	"	1	Square mile,	" S. M.

in.	ft.	yd.	p.	R.	A.	S. M.
144 =	1					
1596 =	9 =	1				
39204 =	272¼ =	30¼ =	1			
1568160 =	10890 =	1210 =	40 =	1		
6272640 =	43560 =	4840 =	160 =	4 =	1	
4014489600 =	27878400 =	3097600 =	102400 =	2560 =	640 =	1

DRY MEASURE.

2	Pints	make	1	Quart,	marked qt.
4	Quarts	"	1	Gallon,	" gal.
2	Gallons	"	1	Peck,	" pk.
4	Pecks	"	1	Bushel,	" bu.
36	Bushels	"	1	Chaldron,	" ch.

pts.		gal.		pk.		bu.		ch.
8	=	1						
16	=	2	=	1				
64	=	8	=	4	=	1		
2304	=	288	=	144	=	36	=	1

This measure is applied to all Dry Goods, as Corn, Fruit, Salt, Coals, &c. A Winchester Bushel is 18½ inches in diameter, and 8 inches deep. The standard Gallon Dry Measure contains 268⅘ inches.

ALE AND BEER MEASURE.

2	Pints	make	1	Quart,	marked qt.
4	Quarts	"	1	Gallon,	" gal.
36	Gallons	"	1	Barrel,	" bar.
54	Gallons	"	1	Hogshead,	" hhd.
2	Hogsheads	"	1	Butt,	" butt.
2	Butts	"	1	Tun,	" tun.

pts.		qt.		gal.		bar.		hhd.		butt.
2	=	1								
8	=	4	=	1						
288	=	144	=	36	=	1				
432	=	216	=	54	=	1½	=	1		
864	=	432	=	108	=	3	=	2	=	1

NOTE. By a law of Massachusetts, the barrel for Cider and Beer shall contain 32 Gallons. The Ale Gallon contains 282 cubic or solid inches.

WINE MEASURE.

4 Gills	make	1 Pint,	marked	pt.
2 Pints	"	1 Quart,	"	qt.
4 Quarts	"	1 Gallon,	"	gal.
42 Gallons	"	1 Tierce,	"	tier.
63 Gallons	"	1 Hogshead,	"	hhd.
2 Tierces	"	1 Puncheon,	"	pun.
2 Hogsheads	"	1 Pipe or Butt,	"	pi.
2 Pipes or 4 Hhds.	"	1 Tun,	"	tun.

pts.		qt.		gal.		tier.		hhd.		pun.		pi.		tun.
2	=	1												
8	=	4	=	1										
336	=	168	=	42	=	1								
504	=	252	=	63	=	1½	=	1						
672	=	336	=	84	=	2	=	1½	=	1				
1008	=	504	=	126	=	3	=	2	=	1½	=	1		
2016	=	1008	=	252	=	6	=	4	=	3	=	2	=	1

NOTE. The Wine Gallon contains 231 cubic inches. We have no statute specifying how many gallons a hogshead, tierce, or pipe, shall contain.

OF TIME.

60 Seconds, or 60″	make	1 Minute,	marked m.
60 Minutes	"	1 Hour,	" h.
24 Hours	"	1 Day,	" d.
7 Days	"	1 Week,	" w.
4 Weeks	"	1 Month,	" mo.
13 Months, 1 day, 6 hours, or 365 days, 6 hours		1 Julian Year,	" y.
12 Calendar months	"	1 Year,	" y.

sec.		m.		h.		d.		w.		mo.		y.
60	=	1										
3600	=	60	=	1								
86400	=	1440	=	24	=	1						
604800	=	10080	=	168	=	7	=	1				
2419200	=	40320	=	672	=	28	=	4	=	1		
31557600	=	525960	=	8766	=	$365\frac{1}{4}$	=		=		=	1

NOTE. The true solar year is the time measured from the sun's leaving either equinox or solstice, to its return to the same again. A periodical year is the time the earth revolves round the sun, and is 365d. 6h. 9m. $14\frac{1}{2}$sec. and is often called the Sidereal year. The civil year is that, which is in common use among the different nations of the world, and contains 365 days for three years in succession, but every fourth year it contains 366 days. When any year can be divided by four, without any remainder, it is leap year, and has 366 days. The days in each month are stated in the following distichs.

Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Except February alone,
Which hath but twenty-eight,
Except leap year, when it hath twenty-nine.

	w.	d.	h.		m.	d.	h.		
Or,	52	1	6	=	13	1	6	=	1 Julian Year.

	day.	h.	m.	sec.		
But,	365	5	48	57	=	1 Solar Year.

	day.	h.	m.	sec.		
And,	365	6	9	$14\frac{1}{2}$	=	1 Sidereal Year.

CIRCULAR MOTION.

60 Seconds	make	1 Prime minute,	marked	'.
60 Minutes	"	1 Degree,	"	°.
30 Degrees	"	1 Sign,	"	s.

12 Signs, or 360 Degrees, the whole great Circle of the Zodiac.

MEASURING DISTANCES.

$7\frac{92}{100}$	Inches	make	1 Link.
25	Links	"	1 Pole.
100	Links	"	1 Chain.
10	Chains	"	1 Furlong.
8	Furlongs	"	1 Mile.

SOLID MEASURE.

1728	Inches	make	1 Foot.
27	Feet	"	1 Yard.
40	Feet of Timber	"	1 Ton.
128	Feet, i. e. 8 feet in length, 4 in height, and 4 in breadth,	"	1 Cord of Wood.

Section 8.

UNITED STATES' MONEY.

ADDITION.

RULE. *Place dollars under dollars, dimes under dimes, cents under cents, and mills under mills, and add the columns together, as in the addition of simple numbers, and place a period or point immediately after the dollars, separating them from the cents.*

NOTE. The eagles and dollars are usually written together; as are also the dimes, cents, and mills. The dollars are separated from the cents by a point; all the figures at the *left* of the point are dollars, and, at the *right* of the point, the first two figures are cents, and the third is mills. Three dollars fifteen cents six mills are written $3.156. Seventy-four dollars three cents four mills are written $74.034. Seventeen dollars five mills are written $17.005.

1.	2.	3.	4.
E. $. d. cts. m.	$. cts. m.	$. cts. m.	$. cts.
7. 5. 6. 4. 3	75.643	16.705	147.86
1. 6. 8. 9. 7	16.897	14.003	789.58
4. 3. 8. 1. 6	43.816	18.719	496.37
5. 8. 3. 1. 3	58.313	97.009	911.34
19. 4. 6. 6. 9	194.669	146.436	2345.15

5. Bought a coat for $17.81; a vest for $3.75; a pair of pantaloons for $2.87; and a pair of boots for $7.18; what was the amount? Ans. 31.61.

6. Sold a load of wood for seven dollars six cents; five bushels of corn for four dollars seventy-five cents, and seven bushels of potatoes for two dollars six cents; what was received for the whole? Ans. $ 13.87.

SUBTRACTION.

	7.	8.	9.	10.
	$. cts. m.	$. cts.	$. cts. m.	$. cts.
From	61.585	471.81	156.003	141.70
Take	19.197	158.19	19.009	90.91
	$42.388	$313.62	$136.994	$50.79

11. From $ 71.07 take $ 5.09. Ans. $ 65.98.
12. From $ 100. take $ 17.17. Ans. $ 82.83.
13. Bought a horse for one hundred seventy-five dollars, and sold him for two hundred twenty-nine dollars eight cents; what was gained by the bargain?
Ans. $ 54.08.
14. From one hundred dollars, there was paid to one man seventeen dollars nine cents, to another twenty-three dollars eight cents, and to another thirty-three dollars twenty-five cents; how much cash remained?
Ans. $ 26.58.
15. From ten dollars take nine mills. Ans. $ 9.991.

MULTIPLICATION.

RULE. *Multiply the quantity by the price, and in the answer point off as many figures for cents and mills, as there are in the price.*

16. What cost 143 barrels of flour at $ 7.25 per barrel?
Ans. 1036.75.

OPERATION.

143
7.25
715
286
1001
$1036.75 Ans.

17. What cost 144 gallons of oil at $1.625 a gallon?

OPERATION. Ans. $234.00.

144
1.625
720
288
864
144
$234.000 Ans.

18. What will 165 gallons of molasses cost at $0.27 a gallon? Ans. $44.55.

19. Sold 73 tons of timber at $5.68 a ton; what was the amount? Ans. $414.64.

20. What cost 43 rakes at $.17 a piece? Ans. $7.31.

21. What cost 19 bushels of salt at $1.625 per bushel? Ans. $30.875.

22. What cost 47 acres of land at $37.75 per acre? Ans. $1774.25.

23. What cost 19 dozen penknives at $.375 a piece? Ans. $85.50.

24. What is the value of 17 chests of souchong tea, each weighing 59 pounds, at $.67 per pound? Ans. $672.01.

25. When 19 cords of wood are sold at $5.63 per cord; what is the amount? Ans. $106.97.

26. A merchant sold 18 barrels of pork, each weighing 200 pounds, at 12 cents 5 mills a pound; what did he receive? Ans. $450.00.

27. A farmer sold one lot of land, containing 187 acres, at $37.50 per acre; another lot, containing 89 acres, at $137.37 per acre; and another lot, containing 57 acres, at $89.29 per acre; what was the amount received for the whole? Ans. $24327.96.

DIVISION.

RULE. *Divide the price by the quantity, or divide the dollars and cents by the number of things either bought or sold, and the quotient will be the answer, which must be pointed off like the dividend.*

28. If 59 yards of cloth cost $90.27, what cost one yard? Ans. $1.53.

OPERATION.

59)90.27(1.53
59
312
295
177
177

29. If 89 acres of land cost $12225.93, what is the value of one acre? Ans. $137.37.

30. When 19 yards of cloth are sold for $106.97, what should be paid for one yard? Ans. $5.63.

31. Gave $22.50 for 18 barrels of apples; what was paid for 1 barrel? For 5 barrels? For 10 barrels? Ans. $20.00 for all.

32. Bought 153 pounds of tea for $90.27; what was it per pound? Ans. $0.59.

33. A merchant purchased a bale of cloth containing 73 yards, for $414.64; what was the cost of one yard? Ans. $5.68.

Section 9.

COMPOUND ADDITION.

COMPOUND ADDITION is the adding together of two or more numbers of different denominations.

1. Paid a London tailor £7. 13s. 6d. 2qr. for a coat, £2. 17s. 9d. 1qr. for a vest, £3. 8s. 3d. 3qr. for pantaloons, and £9. 11s. 8d. 3qr. for a surtout; what was the amount of the bill? Ans. £23. 11s. 4d. 1qr.

OPERATION.

£.	s.	d.	qr.
7	13	6	2
2	17	9	1
3	8	3	3
9	11	8	3
23	11	4	1

The sum of the farthings in the right hand column is 9 farthings, equal to 2d. 1qr.; we write the farthings under the column farthings, and carry the 2d. to the column of pence, the sum of which is 28d. equal to 2s. 4d.; we write the

4d. under its proper column, and add the 2s. to the column of shillings, the sum of which is 71s. equal to £3. 11s.; having written the 11s., we add the £3 to its column, and find the sum of which to be £23. From the above process, we induce the following

RULE.

Write all the given numbers of the same denomination under each other; then add the numbers of the lowest denomination together, and divide their sum by so many as make one of the next higher denomination; set the remainder under its column, and add the quotient to the next column; which add together and divide as before; thus proceed to the last denomination, under which place its whole sum.

2. What is the sum of £6. 19s. 11d. 3qr., £9. 6s. 3d. 3qr., £13. 18s. 3d. 1qr., and £67. 0s. 8d. 1qr.?

Ans. £97. 5s. 3d. 0qr.

TROY WEIGHT.

3.

lbs.	oz.	dwt.	gr.
15	11	19	22
71	10	13	17
65	9	17	14
73	11	13	13
14	8	9	9
242	4	14	3

4.

lbs.	oz.	dwt.	gr.
10	10	10	10
81	11	19	23
47	7	8	19
16	9	10	14
33	10	9	21

APOTHECARIES' WEIGHT.

5.

lb.	℥.	ʒ.	℈.	gr.
81	11	6	1	19
75	10	7	2	13
14	9	7	1	12
37	8	1	1	11
61	11	3	2	3
272	4	3	0	18

6.

lb.	℥.	ʒ.	℈.	gr.
35	9	6	2	19
71	1	1	1	11
37	3	3	2	12
14	4	7	1	13
75	5	6	1	17

AVOIRDUPOIS WEIGHT.

7.

Ton.	cwt.	qr.	lb.	oz.	dr.
71	19	3	27	14	13
14	13	1	11	13	12
39	9	3	13	9	9
15	17	3	16	10	14
61	16	3	13	7	8
203	17	3	27	8	8

8.

Ton.	cwt.	qr.	lb.	oz.	dr.
14	13	2	15	15	15
13	17	3	13	11	13
46	16	3	11	13	10
14	15	2	7	6	9
11	17	3	16	15	11

LONG MEASURE.

9.

deg.	m.	fur.	rd.	ft.	in.
18	19	7	15	11	1
61	47	6	39	10	11
78	32	5	14	9	9
17	59	7	36	16	10
28	56	1	30	16	1
205	8	1	17	15	2

10.

m.	fur.	rd.	yd.	ft.	in.
12	7	35	5	2	11
13	6	15	3	1	10
16	1	17	1	2	5
13	4	13	2	1	9
17	7	36	5	2	7

LAND OR SQUARE MEASURE.

11.

A.	R.	p.	ft.	in.
67	3	39	272	143
78	3	14	260	116
14	2	31	167	135
67	1	17	176	131
49	3	31	69	117
278	3	15	131	102

12.

A.	R.	p.	yd.	ft.	in.
43	1	15	30	8	17
16	3	39	19	7	141
47	1	16	27	5	79
38	3	17	18	8	17
15	1	32	11	1	117

CLOTH MEASURE.

13.

yd.	qr.	na.	in.
5	3	3	2
7	1	1	2
8	3	3	1
9	1	2	2
4	3	3	2
36	3	0	0

14.

E. E.	qr.	na.	in.
16	3	2	1
71	1	1	2
13	3	2	1
47	3	2	2
39	2	3	2

SOLID MEASURE.

15.

Ton.	ft.	in.
17	39	1371
61	17	1711
47	16	1666
71	38	1711
47	17	1617
246	11	1164

16.

Cord.	ft.	in.
14	116	1169
67	113	1711
96	127	969
19	98	1376
14	37	1414

WINE MEASURE.

17.

Tun.	hhd.	gal.	qt.	pt.
61	1	62	3	1
71	3	14	1	1
60	0	17	3	0
14	1	51	1	1
57	3	14	3	1
265	2	35	1	0

18.

Tun.	hhd.	gal.	qt.	pt.
14	3	18	3	0
81	1	60	3	1
17	3	61	3	0
61	3	57	3	1
17	1	17	1	0

ALE AND BEER MEASURE.

19.

Tun.	hhd.	gal.	qt.	pt.
15	3	50	3	1
67	3	17	3	1
17	1	44	1	0
71	3	12	3	1
81	1	18	1	0
254	1	36	0	1

20.

Tun.	hhd.	gal.	qt.	pt.
67	1	51	1	0
15	3	16	3	1
44	1	45	1	1
15	2	12	2	1
67	3	35	1	0

DRY MEASURE.

21.

Ch.	bu.	pk.	qt.	pt.
15	35	3	7	1
61	16	3	6	1
51	30	1	5	0
42	17	2	2	1
14	14	1	4	1
186	7	1	2	0

22.

Ch.	bu.	pk.	qt.	pt.
71	17	1	1	1
16	31	3	3	0
41	14	3	1	1
71	17	1	0	1
10	10	2	3	0

TIME.

23.

y.	d.	h.	m.	s.
57	300	23	59	17
47	169	15	17	38
29	364	23	42	17
18	178	16	38	47
49	317	20	52	57
203	237	4	30	56

24.

w.	d.	h.	m.	s.
15	6	23	15	17
61	5	15	27	18
71	6	21	57	58
18	5	19	39	49
87	6	19	18	57

CIRCULAR MOTION.

25.

S.	°	′	″
11	28	56	58
10	21	51	37
8	13	39	57
8	19	38	49
7	17	47	48
11	11	55	09

26.

S.	°	′	″
6	17	17	18
7	09	19	51
8	18	57	45
4	17	16	39
7	27	38	48

MEASURING DISTANCES.

27.

m.	fur.	ch.	p.	l.
17	5	8	3	24
16	3	7	1	21
47	7	9	3	19
19	6	6	1	16
31	7	1	0	20
133	7	4	0	00

28.

m.	fur.	ch.	p.	l.
14	7	9	3	21
37	1	0	3	16
17	7	8	3	17
61	6	5	3	16
47	1	1	0	23

Section 10.

COMPOUND SUBTRACTION.

COMPOUND SUBTRACTION teaches to find the difference between two numbers of different denominations.

1. A bill on the bank of England for £713. 17s. 11d. 3qr. was sold for £765. 16s. 10d. 1qr.; what was the sum gained? Ans. £51. 18s. 10d. 2qr.

OPERATION.

	£	s.	d.	qr.
From	765	16	10	1
Take	713	17	11	3
Ans.	51	18	10	2

In performing this question, we cannot take 3qr. from 1qr. but we can *borrow*, as in simple numbers, 1 penny = 4qr., which we add to the 1qr. = 5qr. Take 3qr. from 5qr., and 2qr. remain, which we write under the column of farthings; and, as in simple numbers, we carry *one* to the next lower number before subtracting. And 1d. carried to 11d. is 12d.; but, as we cannot take 12d. from 10d., we must again borrow 1s. from the 16s. = 12d. and add it to the 10d. = 22d. Then take 12d. from 22d. = 10d., which we set down and carry one, as before, and so on till the whole be subtracted. Hence the following

RULE.

Write those numbers under each other, which are of the same denomination, the less compound number under the greater. Begin with the lowest denomination, and subtract each lower number from the one above it, and write the difference underneath. If any lower number be larger than the upper, suppose as many to be added to the upper number as would make one of the next higher denomination, then subtract the lower figure, remembering to carry one *to the next lower number before subtracting it; and proceed thus, till all the numbers are subtracted.*

2. From £87. 11s. 9d. 3qr. take £41. 5s. 6d. 1qr.
Ans. £46. 6s. 3d. 2qr.

TROY WEIGHT.

3.

lb.	oz.	dwt.	gr.
15	3	12	14
9	11	17	21
5	3	14	17

4.

lb.	oz.	dwt.	gr.
711	1	3	17
19	3	18	19

APOTHECARIES' WEIGHT.

5.

lb.	℥.	ʒ.	℈.	gr.
15.	7	1	2	15
11	9	7	1	19
3	9	2	0	16

6.

lb.	℥.	ʒ.	℈.	gr.
161	6	3	1	17
97	7	1	2	18

AVOIRDUPOIS WEIGHT.

7.

T.	cwt.	qr.	lb.	oz.	dr.
117	16	1	13	0	14
19	17	3	27	1	15
97	18	1	13	14	15

8.

T.	cwt.	qr.	lb.	oz.	dr.
11	1	0	1	1	13
9	18	3	1	13	15

CLOTH MEASURE.

9.

yd.	qr.	na.	in.
15	1	1	2
9	3	3	1
5	1	2	1

10.

E. E.	qr.	na.	in.
171	2	2	1
19	3	0	2

LONG MEASURE.

11.

deg.	m.	fur.	rd.	yd.	ft.	in.
97	3	7	31	1	1	3
19	17	1	39	1	2	7
77	56	1	31	5	0	2

12.

deg.	m.	fur.	rd.	ft.	in.
18	19	1	1	3	7
9	28	7	1	16	9

LAND OR SQUARE MEASURE.

13.

A.	R.	p.	ft.	in.
116	1	13	100	113
87	3	17	200	117
28	1	35	172	32

14.

A.	R.	p.	yd.	ft.	in.
139	1	17	18	1	30
97	3	18	30	1	31

SOLID MEASURE.

15.

Tons.	ft.	in.
171	30	1000
98	37	1234
72	32	1494

16.

Cord.	ft.	in.
571	18	1234
199	19	1279

WINE MEASURE.

17.

Tun.	hhd.	gal.	qt.	pt.	gl.
171	3	8	1	1	1
99	1	19	3	1	3
72	1	51	1	1	2

18.

Tun.	hhd.	gal.	qt.	pt.	gl.
71	1	1	1	1	1
9	3	3	3	1	3

ALE AND BEER MEASURE.

19.

Tun.	hhd.	gal.	qt.	pt.
15	1	17	1	0
9	3	19	3	1
5	1	51	1	1

20.

Tun.	hhd.	gal.	pt.	qt.
79	2	2	2	0
19	3	13	3	1

DRY MEASURE.

21.

Ch.	bu.	pk.	qt.	pt.
716	1	2	1	0
19	9	3	1	1
696	27	2	7	1

22.

Ch.	bu.	pk.	qt.	pt.
73	13	3	0	1
19	18	1	3	1

TIME.

23.

y.	d.	h.	m	s.
375	15	13	17	5
199	137	15	1	39
175	242	22	15	26

24.

w.	d.	h.	m.	s.
14	1	3	4	15
9	6	17	37	48

CIRCULAR MOTION.

25.

S.	°	′	″
11	7	13	15
9	29	17	36
1	7	55	39

26.

S.	°	′	″
1	23	37	39
9	15	38	47

MEASURING DISTANCES.

27.

M.	fur.	ch.	p.	l.
21	3	5	2	17
9	5	8	1	20
11	5	7	0	22

28.

M.	fur.	ch.	p.	l.
31	7	1	1	19
18	1	7	3	23

Section 11.

EXERCISES IN COMPOUND ADDITION AND SUBTRACTION.

1. What is the amount of the following quantities of gold; 4lb. 8oz. 13dwt. 8gr., 5lb. 11oz. 19dwt. 23gr., 8lb. 0oz. 17dwt. 15gr., and 18lb. 9oz. 14dwt. 10gr.?
Ans. 37lb. 7oz. 5dwt. 8gr.

2. An apothecary would mix 7℔. 3℥. 2ℨ. 2℈. 1gr. of rhubarb, 2℔. 10℥. 0ℨ. 1℈. 13gr. of cantharides, and 2℔. 3℥. 7ℨ. 2℈. 17gr. of opium; what is the amount of the compound? Ans. 12℔. 5℥. 3ℨ. 0℈. 11gr.

3. Add together 17T. 11cwt. 3qr. 11lb. 12oz., 11T. 17cwt. 1qr. 19lb. 11oz., 53T. 19cwt. 1qr. 17lb. 8oz., 27T. 19cwt. 3qr. 18lb. 9oz., and 16T. 3cwt. 3qr. 0lb. 13oz.
Ans. 127T. 12cwt. 1qr. 12lb. 5oz.

4. A merchant has 3 pieces of cloth; the first contains 37yd. 3qr. 3na., the second 18yd. 1qr. 3na., and the third 31yd. 1qr. 2na.; what is the whole quantity?

Ans. 87yd. 3qr. 0na.

5. Sold 3 loads of hay; the first weighed 2T. 13cwt. 1qr. 17lb., the second 3T. 27lb., and the third 1T. 3qr. 11lb.; what did they all weigh?

Ans. 6T. 14cwt. 1qr. 27lb.

6. What is the sum of the following distances; 16m. 7fur. 18r. 14ft. 11in., 19m. 1fur. 13r. 16ft. 9in., 97m. 3fur. 27r. 13ft. 3in., and 47m. 5fur. 37r. 13ft. 10in?

Ans. 181m. 2fur. 18r. 9ft. 3in.

7. A gentleman has three farms, the first contains 169A. 3R. 15p. 227ft., the second 187A. 1R. 15p. 165ft., and the third 217A. 2R. 28p. 165ft.; what is the whole quantity? Ans. 574A. 3R. 20p. 12½ft.

8. There are 3 piles of wood, the first contains 18cords, 116ft. 1000in., the second 17cords, 111ft. 1600in., and the third 21cords, 109ft. 1716in.; how much in all?

Ans. 58cords, 82ft. 860in.

9. John Thomson has four casks of molasses, the first contains 167gal. 3qt. 1pt., the second 186gal. 1qt. 1pt., the third 108gal. 2qt. 1pt., and the fourth 123gal. 3qt. 0pt.; how much is the whole quantity?

Ans. 586gal. 2qt. 1pt.

10. Add together 17bu. 1pk. 7qt. 1pt., 18bu. 3pk. 2qt., 19bu. 1pk. 3qt. 1pt., and 51bu. 3pk. 0qt. 1pt.

Ans. 107bu. 1pk. 5qt. 1pt.

11. James is 13y. 4m. 13da. old, Samuel is 12y. 11m. 23da., and Daniel is 18y. 9m. 29da.; what is the sum of their united ages? Ans. 45y. 2mo. 5da.

12. Add together 18y. 345da. 13h. 37m. 15s., 87y. 169da. 12h. 16m. 28s., 316y. 144da. 20h. 53m. 18s., and 13y. 360da. 21h. 57m. 15s.

Ans. 436y. 290da. 20h. 44m. 16s.

13. Venus is 3S. 18°. 45′. 15″. east of the sun, Mars is 7S. 15°. 36′. 18″. east of Venus, and Jupiter is 5S. 21°. 38′. 27″. east of Mars; how far is Jupiter east of the sun? Ans. 4S. 26°.

14. A merchant owes a debt in London amounting to £7671, what remains due after he has paid £1728. 17s. 9d.? Ans. £5942. 2s. 3d.

15. From 73lb. of silver there was made 26lb. 11oz. 13dwt. 14gr. of plate; what quantity remained?
Ans. 46lb. 0oz. 6dwt. 10gr.

16. From 71℔. 8℥. 13. 1℈. 14gr. take 7℔. 9℥. 13. 1℈. 17gr. Ans. 63℔. 10℥. 7℥. 2℈. 17gr.

17. From 28T. 13cwt. take 10T. 17cwt. 19lb. 14oz.
Ans. 17T. 15cwt. 3qr. 8lb. 2oz.

18. From 76yd. take 18yd. 3qr. 2na.
Ans. 57yd. 0qr. 2na.

19. From 20m. take 3m. 4fur. 18r. 13ft. 8in.
Ans. 16m. 3fur. 21r. 2ft. 10in.

20. From 144A. 3R. take 18A. 1R. 17p. 200ft. 100in.
Ans. 126A. 1R. 22p. 71ft. 80in.

21. From 18 cords take 3 cords 100ft. 1000in.
Ans. 14 cords. 27ft. 728in.

22. From 17T. take 5T. 18ft. 765in.
Ans. 11T. 21ft. 963in.

23. From 169gal. take 76gal. 3qt. 1pt.
Ans. 92gal. 0qt. 1pt.

24. From 17Ch. 18bu. take 5Ch. 20bu. 1pk. 7qt.
Ans. 11Ch. 33bu. 2pk. 1qt.

25. From 83y. take 47y. 10mo. 27da. 18h. 50m. 14s.
Ans. 35y. 1mo. 2da. 5h. 9m. 46s.

26. From 11S. 15°. 36′. 15″. take 5S. 18°. 50′. 18″.
Ans. 5S. 26°. 45′. 57″.

27. A carpenter sent two of his apprentices to ascertain the length of a certain fence. The first stated it was 17r. 16ft. 11in., the second said it was 18r. 5in. The carpenter finding a discrepancy in their statements, and fearing they might both be wrong, ascertained the true length himself, which was 17r. 5yd. 1ft. 11in.; how much did each differ from the other? Ans.

28. From a mass of silver, weighing 106lb., a goldsmith made 36 spoons, weighing 5lb. 11oz. 12dwt. 15gr., a tankard, 3lb. 0oz. 13dwt. 14gr., a vase, 7lb. 11oz. 14dwt. 23gr.; how much unwrought silver remains?
Ans. 88lb. 11oz. 18dwt. 20gr.

29. From a piece of cloth, containing 17yd. 3qr., there were taken two garments, the first measuring 3yd. 3qr. 2na., the second 4yd. 1qr. 3na.; how much remained?
Ans. 9yd. 1qr. 3na.

30. The longitude of a certain star is 3S. 18°. 14′. 35″.,

and the longitude of Jupiter is 11S. 25°. 30′. 50″.; how far will Jupiter have to move in his orbit to be in the same longitude of the star?

Ans. 3S. 22°. 43′. 45″.

Section 12.

REDUCTION.

MENTAL OPERATIONS.

1. In 2 pence how many farthings? In 4 pence? In 5 pence? In 7 pence? In 8 pence? In 10 pence?
2. How many pence in 8 farthings? In 12 farthings? In 16 farthings? In 24 farthings? In 36 farthings?
3. In 2 shillings how many pence? In 4 shillings? In 5 shillings? In 6 shillings? In 7 shillings?
4. In 4 yards how many quarters? In 5 yards? In 6 yards? In 7 yards? In 8 yards? In 9 yards?
5. In 8 quarters how many yards? In 12 quarters? In 16 quarters? In 24 quarters? In 32 quarters?
6. In 3 feet how many inches? In 5 feet? In 7 feet? In 8 feet? In 9 feet? In 10 feet? In 12 feet?
7. In 36 inches how many feet? In 48 inches? In 60 inches? In 72 inches? In 96 inches? In 144 inches?
8. In 6 feet how many yards? In 9 feet? In 12 feet? In 21 feet? In 24 feet? In 30 feet? In 36 feet?
9. In 4 yards how many feet? In 3 yards? In 7 yards? In 9 yards? In 10 yards? In 11 yards? In 12 yards?
10. In 2 acres how many roods? In 3 acres? In 4 acres? In 6 acres? In 7 acres? In 10 acres?
11. In 12 roods how many acres? In 8 roods? In 16 roods? In 20 roods? In 32 roods? In 36 roods?
12. How many furlongs in 2 miles? In 3 miles? In 6 miles? In 7 miles? In 8 miles? In 10 miles?
13. In 12 furlongs how many miles? In 16 furlongs? In 40 furlongs? In 44 furlongs? In 96 furlongs?
14. In 5 dimes how many cents? In 6 dimes? In 8 dimes? In 9 dimes? In 10 dimes? In 12 dimes?

15. How many dimes in 20 cents? In 30 cents? In 40 cents? In 80 cents? In 90 cents? In 100 cents?
16. How many square feet in 1 yard? In 2 yards? In 3 yards? In 5 yards? In 7 yards? In 8 yards?
17. In 9 square feet how many square yards? In 27 feet? In 36 feet? In 54 feet? In 63 feet? In 108 feet?
18. In 1 gallon how many quarts? In 3 gallons? In 5 gallons? In 7 gallons? In 8 gallons? In 9 gallons?
19. How many gallons in 4 quarts? In 8 quarts? In 16 quarts? In 24 quarts? In 32 quarts? In 40 quarts?
20. How many days in 2 weeks? In 4 weeks? In 5 weeks? In 7 weeks? In 9 weeks? In 10 weeks?
21. In 14 days how many weeks? In 21 days? In 28 days? In 35 days? In 42 days? In 56 days?
22. How many pecks in 1 bushel? In 3 bushels? In 4 bushels? In 6 bushels? In 7 bushels? In 9 bushels?
23. In 8 pecks how many bushels? In 12 pecks? In 16 pecks? In 24 pecks? In 32 pecks? In 40 pecks?
24. If in 1 pound of gold there are 12 ounces, how many ounces in 3 pounds? In 4 pounds? In 6 pounds?
25. In 24 ounces how many pounds? In 36 ounces? In 40 ounces? In 60 ounces? In 84 ounces?
26. In 24 pence how many shillings? In 36 pence? In 48 pence? In 60 pence? In 72 pence? In 144 pence?

The student will now perceive, that the object of

REDUCTION is the changing of numbers of one denomination to another without losing their value.

It consists of two parts, Descending and Ascending. The former is performed by Multiplication, and the latter by Division.

Reduction Descending teaches to bring numbers of a higher denomination to a lower; as, to bring pounds into shillings, or tons into hundred-weights.

Reduction Ascending teaches to bring numbers of a lower denomination into a higher; as, to bring farthings into pence, or shillings into pounds.

Section 13.

REDUCTION DESCENDING.

1. In 16cwt. 3qr. 18lb. how many pounds?

Ans. 1894lb.

OPERATION.

Cwt.	qr.	lb.
16	3	18
4		
64		
3		
67		
28		
536		
134		
1876		
18		
1894.		

In this question, we multiply the 16cwt. by 4, because it takes 4 quarters to make one hundred weight; and to this product we add the 3qr. in the question. Then we multiply by 28, because it takes 28 pounds to make one quarter, and to the product we add the 18 pounds in the question, and our work is done.

From the above illustration, we deduce the following

RULE.

Multiply the highest denomination given by so many of the next less, as will make one of that greater; and so proceed until it is brought to the denomination required, observing to bring in the lower denominations to their respective places.

Note 1. To multiply by a ½, we divide the multiplicand by 2; and to multiply by a ¼, we divide by 4.

Note 2. The answers to Reduction Descending will be found in the questions of Reduction Ascending.

2. In £379 how many farthings?
3. In £46. 18s. 5d. how many pence?
4. How many grains Troy in 37lb.
5. In 17lb. of calomel how many grains?
6. In 15 tons how many ounces?
7. In 17cwt. 3qr. 19lb. how many pounds?
8. How many quarters in 144 yards?
9. How many nails in 57 Ells English?

10. How many rods in 97 miles ?
11. How many inches in 7 furlongs ?
12. In 95,000,000 miles how many inches ?
13. In 48deg. 18m. 7fur. 18r. how many feet ?
14. How many square feet in 76 acres ?
15. How many square yards in 144 acres ?
16. How many square inches in 25 square miles ?
17. How many square feet in 7A. 3R. 16p. 218ft ?
18. In 15 tons of timber how many cubic inches ?
19. How many cubic inches in 19 cords, 116 feet ?
20. In 7 hogsheads of wine how many pints ?
21. In 5hhd. 17gal. 3qt. how many quarts ?
22. In 17hhd. of beer how many pints ?
23. How many pints in 57 bushels ?
24. How many quarts in 15Ch. 16bu. 3pk. ?
25. In 57 days how many minutes ?
26. In 365da. 6h. how many seconds ?
27. In 1842 years (365da. 6h. each) how many hours ?
28. In 8S. 14°. 18′. 17″. how many seconds ?

Section 14.

REDUCTION ASCENDING.

1. In 1894lb. how many hundred weight ?

OPERATION.

28)1894 lbs.
4)67. 18lbs.
16cwt. 3qr. 18lb. Ans.

Ans. 16cwt. 3qr. 18lb.

We first divide by 28, because it takes 28lb. to make a quarter of a hundred weight. We then divide by 4, because it takes 4 quarters to make one hundred weight. Hence the following

RULE.

Divide the lowest denomination given by that number, which it takes of that denomination to make one of the next higher; so proceed until it is brought to the denomination required.

NOTE 1. To divide by $5\frac{1}{2}$, or $16\frac{1}{2}$, reduce both divisors and dividends to *halves* by multiplying by 2. To divide by $272\frac{1}{4}$, reduce the number to *fourths* by multiplying by 4. If there be a remainder, it will be *halves* or *fourths*, like the dividend.

NOTE 2. The answers to Reduction Ascending are the questions in Reduction Descending.

2. In 363840 farthings how many pounds ?
3. In 11261 pence how many pounds ?
4. In 213120 grains Troy how many pounds ?
5. In 97920 grains how many pounds, Apothecaries' weight ?
6. In 537600 ounces how many tons ?
7. How many hundred weight in 2007 pounds ?
8. How many yards in 576 quarters ?
9. How many ells English in 1140 nails ?
10. How many miles in 31040 rods ?
11. How many furlongs in 55440 inches ?
12. How many miles in 6,019,200,000,000 inches ?
13. How many degrees in 17714037 feet ?
14. In 3310560 feet how many acres ?
15. How many acres in 696960 square yards ?
16. How many square miles in 100362240000 sq. in. ?
17. How many acres in 342164 square feet ?
18. How many tons of timber in 1036800 cubic inches ?
19. How many cords of wood in 4402944 cubic inches ?
20. In 3528 pints of wine how many hogsheads ?
21. In 1331 quarts of wine how many hogsheads ?
22. In 7344 pints of beer how many hogsheads ?
23. How many bushels in 3648 pints ?
24. How many chaldrons in 17816 quarts ?
25. How many days in 82080 minutes ?
26. How many days in 31557600 seconds ?
27. How many years in 16146972 hours ?
28. In 915497″ how many signs ?

Section 15.

MISCELLANEOUS.

QUESTIONS TO EXERCISE THE FOREGOING RULES.

1. At $5 per ream, how many reams can be bought for $175? Ans. 35 reams.
2. At $7.50 per barrel, how many barrels of flour can be obtained for $217.50? Ans. 29 barrels.
3. At $75 per ton, how many tons of iron can be purchased for $4875? Ans. 65 tons.
4. At $4 per yard, how many yards of cloth can be bought for $1728? Ans. 432 yards.
5. If a ton of coals cost $8.40, what cost one cwt.? Ans. 42 cents.
6. At $2.40 per bu., what cost 1 pk.? What cost 17bu. 3pk. Ans. $42.60.
7. At $3.50 per quintal, what cost 37 quintals? Ans. $129.50.
8. John Webster bought 5cwt. 3qr. 18lb. of sugar at 9 cents per lb., for which he paid 25 barrels of apples at $1.75 per barrel; how much remains due? Ans. $15.83.
9. If 97lb. of beef cost $8.73, what cost 1lb.? What cost 147lb.? Ans. $13.23.
10. If a man travel 45 miles in 9 hours, how far will he travel in 1 hour? How far in 59 hours? Ans. 295 miles.
11. If a ton of hay can be purchased for $18.40, what will be the price of 1cwt.? What of 47cwt.? Ans. $43.24.
12. Bought 65 barrels of flour for $422.50, what cost one barrel? What cost 15 barrels? Ans. $97.50.
13. For 45 acres of land, a farmer paid $2025; what cost one acre? What 180 acres? Ans. $8100.00.
14. For 5 pairs of gloves, a lady paid $3.45; what cost 1 pair? What cost 11 pairs? Ans. $7.59.
15. When $1480 are paid for 25 acres of land, what cost 1 acre? What cost 1 rod? What cost 37A. 2R. 18p. Ans. $2226.66.

16. Paid $10.08 for 144lb. of pepper; what was the price of one pound? What cost 359lb.?
Ans. $25.13.

17. Paid $77.13 for 857lb. of rice; what cost 1lb.? What cost 359lb.? Ans. $32.31.

18. J. Johnson paid $187.53 for 987gal. of molasses? what cost 1gal.? What cost 329gal.? Ans. $62.51.

19. For 47 bushels of salt, J. Ingersoll paid $26.32; what cost 1 bushel? What cost 39 bushels?
Ans. $21.84.

20. If 15 men can perform a piece of work in 10 days, how long would it take one man to perform the same labor? How long 75 men? Ans. 2 days.

21. A certain field will pasture 10 horses 9 weeks; how long will it pasture 1 horse? How long 18 horses?
Ans. 5 weeks.

22. If a mechanic, by laboring 9 hours per day, can perform a certain piece of work in 10 days, how long would it take him by laboring one hour per day? How long by 15 hours per day? Ans. 6 days.

23. Bought a silver tankard, weighing 2lb. 7oz. for $46.50; what did it cost per oz.? How much per lb.?
Ans. $18.00.

24. Bought 3T. 1cwt. 18lb. of leather at 12 cents per lb., and sold it at 9 cents per lb.; what did I lose?
Ans. $205.50.

25. Phineas Bailey has agreed to grade a certain railroad at $5.75 per rod; what will he receive for grading a road between two cities, whose distance from each other is 37m. 7fur. 29r.? Ans. $69856.75.

26. Bought a hogshead of molasses, containing 100 gallons, for $25; but 15gal. 3qt. having leaked out, I sold the remainder at 12 cents a quart; what did I gain?
Ans. $15.44.

27. From a large farm, containing 765A. 3R. 14p., there were sold 144A. at $75 per acre, and the remainder was sold at $1.67 per square rod; what was the whole amount? Ans. $176954.98.

28. Bought 15T. 3cwt. 15lb. of iron at 6 cents per pound; sold 6T. 1cwt. 1qr. 18lb. at 5 cents per lb., and the remainder at 10 cents per lb.; what did I gain?
Ans. $678.14.

29. John Smith has 3 farms, the first contains 89A. 3R. 39p.; the second 97A. 1R. 15p.; and the third 117A. 1R. 19p. He gave his son 175A. 3R. 29p. and he sold the remainder at $1.25 per square rod. What did he receive? Ans. $25755.00.

30. A lady gave her daughter $10 to go a "shopping"; having purchased 2yd. of silk, at $1.27 per yd., a bonnet for $3.75, 3 pairs of gloves at 0.19 a pair, and two fans at 0.37 each, she returned the remainder of the money to her mother; what was the sum?

Ans. $2.40.

Section 16.

COMPOUND MULTIPLICATION.

MENTAL OPERATIONS.

1. If a penknife cost 9d., what will 2 penknives cost? What will 3? What will 4?

2. If a yard of cloth cost 1s. 6d., what will 2yd. cost? 4yd.? 6 yd.? 7yd.?

3. A boy bought a top, for 1s. 2d.; what will 3 tops cost? What will 5 tops cost?

4. If a man walk 7m. 4fur. in 1 day, how far will he walk in 2 days? In 3 days? In 5 days?

5. If a man consume 5lb. 6oz. of meat in 1 week, how much will he require in 3 weeks?

6. If a small book cost 9d., what will 2 books cost? What will 4 books? What will 6 books?

FOR THE SLATE.

1. If an acre of land cost £14. 5s. 8d. 2qr., what will 9 acres cost? Ans. £128. 11s. 4d. 2qr.

OPERATION.

£.	s.	d.	qr.
14	5	8	2
			9
128	11	4	2

In performing this question, we say 9 times 2 farthings are 18 farthings; these farthings, we reduce to pence by dividing them by 4; and we find the result to be 4d.

and 2 farthings remaining. We set down the 2 farthings and carry 4 to the next product. We then say 9 times 8 pence are 72 pence, to these we add the 4 pence, which make 76 pence, which we divide by 12, the number of pence in a shilling, and find the result to be 6 shillings and 4 pence, we set down the pence and carry the 6 shillings to the next product. We then say 9 times 5 shillings are 45 shillings, to these we add the 6 shillings, and the sum is 51 shillings, which are equal to 2 pounds and 11 shillings. We set down the 11 shillings, and carry the 2 pounds to the next product, and then say 9 times 14 pounds are 126 pounds, to these we add the 2 pounds, and the sum is 128 pounds, which we set down under the pounds in the multiplicand, and the work is finished, and the answer is £128. 11s. 4d. 2qr. Hence we perceive, that when the quantity is less than 12, we may adopt the following

RULE.

Multiply each denomination of the compound number, beginning at the lowest, by the multiplier, and carry as in Compound Addition.

2.

£.	s.	d.
5	6	8
		2
10	13	4

3.

£.	s.	d.
19	11	7
		3
58	14	9

4.

£.	s.	d.
25	17	11
		5
129	9	7

5.

£.	s.	d.
18	15	$8\frac{3}{4}$
		6
112	14	$4\frac{1}{2}$

6.

Cwt.	qr.	lb.	oz.
18	3	17	10
			6
113	1	21	12

7.

Ton.	cwt.	qr.	lb.
14	15	3	12
			7
103	11	0	0

8.

Cwt.	qr.	lb.	oz.
19	1	8	15
			8
154	2	15	8

9.

lbs.	oz.	dr.
15	14	13
		9
143	5	5

10.

M.	fur.	rd.	ft.
97	7	14	13
			6
587	4	8	12

11.

Deg.	m.	fur.	rd.
18	12	6	18
			8
145	32	7	24

12.

Rd.	yd.	ft.	in.
23	3	2	9
			9
213	2	0	9

13.

Fur.	rd.	ft.	in.
9	31	16	11
			10
98	0	4	2

NOTE. The answers to the following questions are found in the corresponding numbers in Compound Division.

14. What cost 7 yards of cloth at 18s. 9d. per yard ?

15. If a man travel 12m. 3fur. 29rd. in one day, how far will he travel in 9 days ?

16. If 1 acre produce 2 tons 13cwt. 19lb. of hay, what will 8 acres produce ?

17. If a family consume 49galls. 3qts. 1pt. of molasses in 1 month, what quantity will be sufficient for one year ?

18. John Smith has 12 silver spoons, each weighing 3oz. 17dwt. 14gr., what is the weight of all ?

19. Samuel Johnson bought 7 loads of timber, each measuring 7 tons 37ft. ; what was the whole quantity ?

20. If the moon move in her orbit 13°. 11′. 35″. in 1 day, how far will she move in 10 days ?

21. If 1 dollar will purchase 2℔. 8℥. 7ʒ. 1℈. 10gr. of ipecacuana, what quantity would 9 dollars buy ?

22. If 1 dollar will buy 2A. 3R. 15p. 30yd. 8ft. 100in. of wild land, what quantity may be purchased for 12 dollars ?

23. Joseph Doe will cut 2 cords 97ft. of wood in 1 day ; how much will he cut in 9 days ?

24. If 1 acre of land produce 3ch. 6bu. 2pk. 7qt. 1pt. of corn, what will 8 acres produce ?

II. If the quantity be such as may be resolved into two or more factors, that is, two or more numbers, whose product shall be equal to the quantity, the compound number may be multiplied by 1 of those numbers, and the product by the other, and the last product will be the value of the whole quantity.

25. What cost 24 yards of broadcloth at £2. 7s. 11d. per yard ?

£	s.	d.
2	7	11
		4
9	11	8
		6
57	10	0

In this question, we find the quantity 24 equal to the product of 4 and 6, we therefore multiply the price first by 4, and then that product by 6, and the last product is the answer. Or we might have multiplied first by 6 and then by 4, and the answer would have been the same.

26. What cost 360 tons of iron at £17. 16s. 1d. per ton?

£.	s.	d.
17	16	1
		6
106	16	6
		6
640	19	0
		10
6409	10	0

In this question, we find the factors of 360 to be 6 and 6 and 10, that is, 6 multiplied by 6 are 36, and 36 multiplied by 10 make 360. We then first multiply by 6, and then that product by 6, and then again the last product by 10. The result wolud have been the same, if we had multiplied by 10 first.

27. If a man travel 3m. 7fur. 18rds. in one day, how far would he travel in 30 days?

28. If a load of hay weigh 2 tons 7cwt. 3qrs. 18lb., what would be the weight of 84 similar loads?

29. When it requires 7yds. 3qr. 2na. of silk to make a lady's dress, what quantity would be sufficient to make 72 similar dresses?

30. A tailor has an order from the navy agent to make 132 garments for seamen; how much cloth will it take, supposing each garment to require 3yds. 2qr. 1na.?

III. When the quantity is more than 12, and the number is such, that it cannot be resolved into two or more factors, the better method is to find the factors of a number *nearest* the given number, and having multiplied the compound number by one of these factors, and the product by the other factor, then find the value of the remaining quantity and add it to the last product.

31. If 1 dollar will buy 17lbs. 10oz. 13dr. of beef, how much may be bought for 62 dollars?

lb.	oz.	dr.
17	10	13
		5
88	6	1
		12
1060	8	12
35	5	10
1095	14	6

lb.	oz.	dr.
17	10	13
		2
35	5	10

As 62 is not the product of any two numbers in the multiplication table, we take some convenient number *less* than 62, viz. 60. This we resolve into two factors 5 and 12, and having found the amount of 60 dollars, we then find the quantity 2 dollars will buy, and add this amount to the former, and the sum is the quantity 62 dollars will buy.

32. What cost 97 tons of lead at £2. 17s. 9½d. per ton?
33. If a man travel 17m. 3fur. 19r. 3yd. 2ft. 7in. in one day, how far would he travel in 38 days?
34. If 1 acre will produce 27bu. 3pk. 6qt. 1pt. of corn, what will 98 acres produce?
35. If it require 7yd. 3qr. 2na. to make 1 cloak, what quantity would it require to make 48 cloaks?
36. One ton of iron will buy 13A. 3R. 14p. 18yd. 7ft. 76in. of land; how many acres will 19 tons buy?

Section 17.

COMPOUND DIVISION.

MENTAL OPERATIONS.

1. If 2 yards of cloth cost 3s., what will 1 yard cost?
2. If 3 barrels of apples cost 5s., what cost 1 barrel?
3. If 4hhds. of lime cost 15s., what cost 1hhd.?
4. Divide 9s. equally among 9 boys.
5. Divide 10d. equally among 3 girls.
6. What is a fourth part of 5 gallons;
7. What is a seventh part of 7 gallons?
8. What is a sixth part of 9 gallons?

FOR THE SLATE.

1. If 9 acres of land cost £128. 11s. 8d. 2qr., what is the value of 1 acre? Ans. £14. 5s. 8d. 2qr.

OPERATION.

	£.	s.	d.	qr.
9)	128	11	4	2
	14	5	8	2

Having divided the pounds by 9, we find the quotient to be £14, which we write under £128, and to the £2. remaining (40s.) we add the 11s. in question, and their amount is 51s.; and these 51s. we again divide by 9, and the quotient is 5s., which we write under the 11s. in the question; and to the remainder, 6s., which are 72d., we add the 4d. in the question, and the sum is 76d.; having again divided these by 9, we write the quotient, 8, under the 4d. in the question; and to the remainder, 4d., which is 16qr., we add the 2qr. in the question, and the amount is 18qr., which we again divide by 9, and find the quotient to be 2qr., which we write under the 2qr. in the question. Thus we find our answer to the question to be £14. 5s. 8d. 2qr. Hence the following

RULE.

I. *Divide the highest denomination by the quantity; and if any thing remains, reduce it to the next lower denomination, and continue to divide until it is reduced to the lowest denomination.*

2.

	£.	s.	d.
2)	10	13	4
	5	6	8

3.

	£.	s.	d.
3)	58	14	9
	19	11	7

4.

	£.	s.	d.
5)	129	9	7
	25	17	11

5.

	£.	s.	d.	qr.
6)	112	14	4	2
	18	15	8	3

6.

	Cwt.	qr.	lb.	oz.
6)	113	1	21	12
	18	3	17	10

7.

	Ton.	cwt.	qr.	lb.
7)	103	11	0	0
	14	15	3	12

8.

	Cwt.	qr.	lb.	oz.
8)	154	2	15	8
	19	1	8	15

9.

	lb.	oz.	dr.
9)	143	5	5
	15	14	13

10.

	M.	fur.	rd.	ft.
6)	587	4	8	12

11.

	Deg.	m.	fur.	rd.
8)	145	32	7	24

12.

	Rd.	yd.	ft.	in.
9)	213	2	0	9

13.

	Fur.	rd.	ft.	in.
10)	98	0	4	2

NOTE. The answers to the following questions are found in the corresponding numbers in Compound Multiplication.

14. What cost 1 yard of cloth, when 7yd. can be bought for £6. 11s. 3d. ?
15. If a man, in 9 days, travel 112m. 1fur. 21rd., how far will he travel in 1 day ?
16. If 8 acres produce 21T. 5cwt. 1qr. 12lb. of hay, what will 1 acre produce ?
17. If a family consume in 1 year 598gal. 2qt. of molasses, how much may be necessary for 1 month ?
18. John Smith has 12 silver spoons, weighing 3lb. 10oz. 11dwt. ; what is the weight of each spoon ?
19. Samuel Johnson bought 7 loads of timber, measuring 55T. 19ft. ; what was the quantity in each load ?
20. If the moon, in 10 days, move in her orbit 4S. 11°. 55′. 50″., how far does she move in 1 day ?
21. If ℔ 9 will buy 24lb. 8℥. 3ʒ. 1℈. 10gr. of ipecacuanha, how large a quantity will ℔ 1 purchase ?
22. When ℔ 12 will buy 34A. 0R. 32p. 8yd. 5ft. 48in. of wild land ; how much will ℔ 1 buy ?
23. Joseph Doe will cut 24 cords 105 feet of wood in 9 days ; how much will he cut in 1 day ?
24. When 8 acres of land produce 25Ch. 17bu. 3pk. 4qt. of grain ; what will 1 acre produce ?

When the quantity is a composite number, that is, one which is composed of the product of two or more numbers, we proceed as in the following question.

25. When 24 yards of broadcloth are sold for £57. 10s. 0d., what is the price of 1 yard ? Ans. £2. 7s. 11d.

£.	s.	d.
6)57	10	0
4)9	11	8
£2	7	11

In this question, we find the component parts, or factors, of 24 are 6 and 4 ; that is, 6 multiplied by 4 produces 24. We therefore first divide the price by one of these numbers, and then divide the quotient by the other. From the above process we deduce the following

RULE.

II. *Divide the dividend by one of the component parts, and the quotient thence arising by the other, and the last quotient will be the answer.*

When the quantity is such, that it cannot be resolved

into two or more factors, the question must be performed by Long Division, as in the following question.

26. If 23cwt. of iron cost £171. 1s. 3d. what cost 1cwt.?

Ans. £7. 8s. 9d.

OPERATION.

£	s.	d.	
23)171	1	3	(£7.
161			
10			
20			
23)201			(8s.
184			
17			
12			
23)207			(9d.
207			

In this question we first divide the pounds by 23, and obtain 7 for the quotient, and £10 remaining, we reduce to shillings and annex the 1s. and again divide by 23 and obtain 8s. for the quotient. The remainder, 17s., we reduce to pence and annex the 3d. and again divide by 23, and obtain 9d. for the quotient. Thus we find the answer to be £7. 8s. 9d.

So in similar cases we should divide the highest denomination by the quantity, and if any thing remains, reduce it to the next lower denomination and continue to divide until it is reduced to the lowest denomination.

27. If a man travel 117m. 7fur. 20rd. in 30 days, how far will he travel in 1 day?

28. If 84 loads of hay weigh 201 Tons 4cwt. 2qr. 0lb., what will 1 load weigh?

29. When 72 ladies require 567yd. 0qr. 0na. for their dresses, how many yards will be necessary for 1 lady?

30. When 132 sailors require 470yd. 1qr. of cloth to make their garments, how many yards will be necessary for 1 sailor?

31. If $62 will buy 1095lb. 14oz. 6dr. of beef, how much may be obtained for $1?

32. Paid £280. 5s. 9½d. for 97 tons of lead; what did it cost per ton?

33. If a man travel 662m. 4fur. 28rd. 3yd. 2ft. 2in. in 38 days, how far will he travel in 1 day?

34. When 98 acres produce 2739bu. 1pk. 5qt. of grain, what will 1 acre produce?

35. A tailor made 48 garments from 378 yards of cloth; what quantity would it take to make 1 garment?

36. When 19 tons of iron will purchase 262A. 3R. 37p. 25yd. 1ft. 40in. of land, how much may be obtained for 1 ton?

Section 18.

BILLS.

Haverhill, March 19, 1842.

Mr. William Greenleaf,

Bought of Moses Atwood,

86 Shovels,	at	$ 0.50.
90 Spades,	"	86.
18 Ploughs,	"	11.00.
23 Handsaws,	"	3.50.
14 Hammers,	"	62.
12 Millsaws,	"	12.12.
46 Cwt. Iron,	"	12.00.
		$ 1105.02.

Received payment,

Moses Atwood.

Lowell, June 5, 1842.

Mr. Amos Dow,

Bought of Lord & Greenleaf.

37 Chests Green Tea,	at	$ 23.75.
42 " Black do.	"	17.50.
43 Casks Wine,	"	99.00.
12 Crates Liverpool Ware,	"	175.00.
19 bls. Genessee Flour,	"	7.00.
23 bu. Rye,	"	1.52.
		$ 8138.71.

Received payment,

Lord & Greenleaf,
by James Clark.

Baltimore, July 19, 1842.

Mr. John Kimball,

Bought of Simon Grey,

14 oz. Gum Camphor,	at	$ 0.63.
12 " Laudanum,	"	.88.
23 " Gum Elastic,	"	.62.
16 " Emetic Tartar,	"	1.27.
17 " Cantharides,	"	2.25.
		$ 92.21.

Received payment,

Simon Grey,
by Enoch Osgood.

New York, May 20, 1842.

Dr. John Smith,

Bought of Somes & Gridley,

82 galls. Temperance Wine,		at	$ .75.
89 " Port,	do.	"	.92.
24 pair Silk Gloves,		"	.50.
			$ 155.38.

Received payment,

Somes & Gridley.

Newburyport, March 7, 1842.

Mr. Levi Webster,

Bought of James Frankland,

6 lbs. Chocolate,	at	$ .18.
12 " Flour,	"	.20.
6 pair Shoes,	"	1.80.
30 lbs. Candles,	"	.26.
		$ 22.08.

Received payment,

James Frankland.

Salem, May 13, 1842.

Mr. Noah Webster,

Bought of Ayer, Fitts, & Co.

80	pair Hose,	at	$ 1.20.
17	" Boots,	"	3.00.
19	" Shoes,	"	1.08.
23	" Gloves,	"	.75.
			$ 184.77.

Received payment,

Ayer, Fitts, & Co.
by William Summers.

Baltimore, June 30, 1842.

Mr. Samuel Osgood,

Bought of Stephen Barnwell,

27	Young Readers,	at	$ .20.
10	Greek Lexicons,	"	3.90.
7	Ainsworth's Dictionaries,	"	4.75.
19	Folio Bibles,	"	2.93
20	Testaments,	"	.37.
			$ 140.72.

Received payment,

Stephen Barnwell.

Philadelphia, August 1, 1842.

Mr. Elias Smith,

Bought of Timothy Eaton,

49	yds. Calico,	at	$ .30.
46	" Irish Linen,	"	2.56.
140	ps. Nankin,	"	2.91.
169	yds. Pongee Silk,	"	2.00.
153	" Blue do.	"	1.37.
			$ 1087.47.

Received payment,

Timothy Eaton.

London, June 19, 1842.

Mr. Edward Snow of Lowell, U. S.

Bought of Smith, Davis, & Co.

241	yds.	Red Broadcloth,	at 16s. 4d.
412	"	Blue do.	" 8s. 9d.
510	"	White do.	" 13s. 5¼d.
424	"	Green do.	" 14s. 6¼d.
169	"	Black Velvet,	" 12s. 8½d.
349	"	Black Kerseymere,	" 17s. 6¼d.
648	"	Carpet,	" 14s. 9½d.

£1919. 18s. 9¼d.

Received payment,

Smith, Davis, & Co.
by Thomas Vance.

Section 19.

FRACTIONS.

MENTAL OPERATIONS.

The pupil must carefully commit all the definitions on page 77, before he commences mental operations.

1. If an apple be divided into two equal parts, one of those parts is called a half, and is written thus, $\frac{1}{2}$.
2. If an apple be divided into three equal parts, one of those parts is called a third, and is written thus, $\frac{1}{3}$.
3. Two of those parts are called two thirds, and are written thus, $\frac{2}{3}$.
4. If an orange is divided into four equal parts, one of those parts is called a quarter, and is written thus, $\frac{1}{4}$. Two of those parts are called two fourths, and are written thus, $\frac{2}{4}$, or thus, $\frac{1}{2}$.
5. Three of those parts are called three quarters, and are written thus, $\frac{3}{4}$.
6. One is what part of two? Ans. $\frac{1}{2}$.
7. One is what part of three? Ans. $\frac{1}{3}$.

8. One is what part of four? Ans. $\frac{1}{4}$. What part of 5?
9. Two is what part of 3? Ans. $\frac{2}{3}$.
10. What part of 5 is 2? Is 3? Is 4? Is 6? Is 7?
11. What part of 7 is 2? Is 3? Is 5? Is 6?
12. What part of 11 is 4? Is 5? Is 6? Is 7?
13. What part of 19 is 5? Is 11? Is 13? Is 17?
14. When corn is 7 shillings a bushel, what part of a bushel could you buy for 1s.? For 2s.? For 5s.?
15. When flour is $9 per barrel, what part of a barrel could be bought for $2? For $3? For $7?
16. If $\frac{1}{6}$ of a barrel of flour cost $2, what will $\frac{2}{6}$ cost? What will $\frac{3}{6}$? What will $\frac{4}{6}$? What will $\frac{5}{6}$?
17. If $\frac{1}{7}$ of a cwt. of sugar cost $14, what will $\frac{1}{7}$ cost?
18. What will $\frac{2}{7}$ cost? $\frac{3}{7}$? $\frac{4}{7}$? $\frac{5}{7}$? $\frac{6}{7}$? $\frac{7}{7}$?
19. If $\frac{1}{11}$ of a pound of tea cost 35 cents, what will $\frac{1}{11}$ cost? $\frac{3}{11}$? $\frac{5}{11}$? $\frac{6}{11}$? $\frac{8}{11}$? $\frac{11}{11}$?
20. If $\frac{1}{13}$ of a yard of cloth cost 30 cents, what will $\frac{1}{13}$ cost? What will $\frac{3}{13}$ cost? $\frac{4}{13}$? $\frac{5}{13}$? $\frac{6}{13}$? $\frac{7}{13}$? $\frac{13}{13}$?
21. If $\frac{1}{8}$ of an acre cost $28, what will $\frac{1}{8}$ cost? What will an acre cost?
22. If $\frac{1}{9}$ of a share in a railroad be worth $36, what is $\frac{1}{9}$ worth? What is the value of a whole share?
23. When $\frac{1}{11}$ of a share in a factory cost $60, what is the value of $\frac{1}{11}$? What is the value of a whole share?
24. Gave $21 for $\frac{3}{4}$ of a yard of broadcloth, what cost $\frac{1}{4}$ of a yard? What cost a yard?
25. Webster paid $8 for $\frac{2}{3}$ of a chest of tea; what would $\frac{1}{3}$ of a chest cost? What would $\frac{1}{2}$ of a $\frac{1}{3}$ cost? What $\frac{1}{4}$ of a $\frac{1}{3}$ cost?
26. When $\frac{1}{11}$ of a ton of iron is sold for $32; what is the cost of $\frac{1}{11}$? Of $\frac{1}{2}$ of $\frac{1}{11}$? Of $\frac{1}{4}$ of $\frac{1}{11}$?
27. Peter Jones paid $16 for $\frac{1}{15}$ of an ox; what cost $\frac{1}{15}$ of the ox, and what did Richard Martin pay for $\frac{1}{2}$ of a $\frac{1}{15}$? What did S. Ayer pay for a $\frac{1}{4}$ of a $\frac{1}{15}$?
28. Paid John Atwood $128 for $\frac{1}{5}$ of his farm; what is the value of $\frac{1}{5}$, and what must J. Kimball pay for $\frac{1}{2}$ of a $\frac{1}{5}$? What is the value of the whole farm?
29. D. Webster bought $\frac{1}{3}$ of a saw mill, for which he paid $300. What was the value of the whole mill? What is the value of $\frac{1}{7}$ of the mill? Of $\frac{1}{4}$ of $\frac{1}{7}$? Of $\frac{1}{5}$ of $\frac{1}{4}$ of $\frac{1}{7}$?
30. 15 is $\frac{1}{5}$ of what number? Is $\frac{2}{7}$? Is $\frac{3}{7}$? Is $\frac{3}{11}$?

31. 21 is $\frac{3}{7}$ of what number? Is $\frac{2}{6}$? Is $\frac{3}{10}$? Is $\frac{3}{11}$?
32. 30 is $\frac{6}{11}$ of what number? Is $\frac{6}{7}$? Is $\frac{6}{5}$? Is $\frac{6}{13}$?
33. 14 is $\frac{2}{12}$ of what number? Is $\frac{2}{3}$? Is $\frac{2}{11}$? Is $\frac{2}{5}$?
34. 12 is $\frac{2}{10}$ of what number? Is $\frac{2}{11}$? Is $\frac{2}{7}$? Is $\frac{2}{3}$?
35. 18 is $\frac{9}{11}$ of what number? Is $\frac{9}{10}$? Is $\frac{9}{13}$? Is $\frac{9}{20}$?
36. Samuel Page sold a pair of oxen for $48, which was $\frac{6}{7}$ of their cost. What did he lose?
37. Bought a horse for $72, which was $\frac{8}{9}$ of his real value; what did I gain?
38. 72 is $\frac{8}{9}$ of what number?
39. Sold a quantity of depreciated money for $81, which was $\frac{9}{11}$ of its nominal value; what was the sum sold?
40. Having improved a chaise 15 years, it was sold for $25, which was only $\frac{5}{12}$ of what it cost. What was the original price?
41. A Loafer shot at a flock of pigeons on a tree, and killed 24, which was $\frac{3}{7}$ of the number. How many pigeons will remain on the tree?

Section 20.

VULGAR FRACTIONS.

FRACTIONS are parts of an integer.

VULGAR FRACTIONS are expressed by two terms, called the Numerator and Denominator; the former above, and the latter below a line.

Thus; $\begin{cases}\text{Numerator}\\ \text{Denominator}\end{cases}\ \frac{7}{11}$.

The Denominator shows into how many parts the integer, or whole number, is divided.

The Numerator shows how many of those parts are taken.

1. A proper fraction is one whose numerator is less than the denominator, as $\frac{5}{7}$.
2. An improper fraction is one whose numerator exceeds, or is equal to, the denominator; as $\frac{17}{12}$ or $\frac{8}{8}$.
3. A simple fraction has a numerator and denominator only; as $\frac{8}{9}$, $\frac{4}{7}$.

4. A compound fraction is a fraction of a fraction, connected by the word *of;* as $\frac{7}{8}$ of $\frac{5}{6}$ of $\frac{3}{4}$.
5. A mixed number is an integer with a fraction; as $7\frac{6}{11}$, $5\frac{3}{8}$.
6. A compound mixed fraction is one whose numerator or denominator, or both, is a mixed number; as $\frac{7\frac{1}{2}}{11}$, or $\frac{44}{7\frac{1}{2}}$.
7. The greatest common measure of two or more numbers is the largest number, that will divide them without a remainder.
8. The least common multiple of two or more numbers is the least number, that may be divided by them without a remainder.
9. A fraction is in its lowest terms, when no number but a unit will measure both its terms.
10. A prime number is that which can be measured only by itself or a unit; as 7, 11, and 19.
11. A perfect number is equal to the sum of all its aliquot parts; as 6, 28, 496, &c.
12. A fraction is equal to the number of times the numerator will contain the denominator.
13. The value of a fraction depends on the proportion, which the numerator bears to the denominator.

I. To find the greatest common measure of two or more numbers; that is, to find the greatest number that will divide two or more numbers.

1. What is the common measure of 84 and 132; that is, what is the largest number, that will divide both of these numbers without a remainder? Ans. 12.

OPERATION.

84)132(1
84
48)84(1
48
36)48(1
36
12)36(3
36

As 12 will divide 36, it is evident it will also divide 48, which is equal to 12 + 36. It will also divide 84; because 84 is equal to 36 + 48; for, as 12 will divide each of these numbers, it is evident it will divide their sum. For the same reason, it will also divide 132, which is equal to 84 + 48. We therefore find, that 12 is the largest number, that will di-

vide 48 and 132 without a remainder. It is, therefore, its greatest common measure. Hence the following

RULE.

Divide the greater number by the less, and if there be a remainder, divide the last divisor by it, and so continue dividing the last divisor by the last remainder, until nothing remains, and the last divisor is the greatest common measure.

If there be more than two numbers, find the greatest common measure of two of them, and then of that common measure and the other numbers. If it should happen, that 1 *is the common measure, the numbers are prime to each other, and are incommensurable.*

2. What is the greatest common measure of 85 and 95? Ans. 5.

3. What is the greatest common measure of 72 and 168? Ans. 24.

4. What is the greatest common measure of 119 and 121? Ans. 1.

5. What is the largest number that will divide 324 and 586? Ans. 2.

6. What is the largest number that will divide 582 and 684? Ans. 6.

7. What is the greatest common measure of 32 and 172? Ans. 4.

8. What is the largest number that will divide 84 and 1728? Ans. 12.

9. What is the greatest common measure of 16, 20, and 26? Ans. 2.

10. What is the greatest common measure of 12, 18, 24, and 30? Ans. 6.

II. To reduce fractions to their lowest terms.

NOTE. A fraction is said to be in its lowest terms, when no number but a unit will divide its numerator and denominator.

1. Reduce $\frac{5}{15}$ to its lowest terms.

OPERATION.

$5)\frac{5}{15} = \frac{1}{3}$ Ans.

We find by the last Rule, that 5 is the largest number, that will divide both the numerator and denominator of the fraction; and having divided them both by it, we

find the result to be $\frac{1}{3}$, and that $\frac{1}{3}$ is equal to $\frac{5}{15}$ is evident from the fact, that the ratio of 5 to 15 is equal to the ratio of 1 to 3. And, as the value of a fraction depends on the ratio, which the numerator bears to the denominator, if their ratios are equal, the fractions are also equal. Q. e. d. Hence the following

RULE.

Divide the numerator and denominator by any number that will divide them both without a remainder; and so continue until no number will divide them but unity. Or, divide the numerator and denominator by the greatest common measure.

2. Reduce $\frac{5}{25}$ to its lowest terms. Ans. $\frac{1}{5}$.
3. Reduce $\frac{6}{36}$ to its lowest terms. Ans. $\frac{2}{6}$.
4. Reduce $\frac{12}{36}$ to its lowest terms. Ans. $\frac{1}{3}$.
5. Reduce $\frac{98}{144}$ to its lowest terms. Ans. $\frac{7}{12}$.
6. Reduce $\frac{107}{214}$ to its lowest terms. Ans. $\frac{1}{2}$.
7. Reduce $\frac{123}{369}$ to its lowest terms. Ans. $\frac{123}{369}$.
8. Reduce $\frac{81}{567}$ to its lowest terms. Ans. $\frac{1}{7}$.
9. Reduce $\frac{7891}{81116}$ to its lowest terms. Ans. $\frac{7891}{81116}$.
10. What is the lowest expression of $\frac{346}{618}$? Ans. $\frac{173}{309}$.

III. To reduce mixed numbers to improper fractions.

MENTAL OPERATIONS.

1. In 3 dollars how many halves? How many thirds?
2. In 7 apples how many tenths? How many twelfths?
3. In $8\frac{3}{7}$ dollars how many sevenths?
4. In $3\frac{1}{4}$ oranges how many fourths?
5. In $9\frac{4}{11}$ gallons how many elevenths?
6. In $7\frac{3}{5}$ quarts how many fifths of quarts?

OPERATION.

7
5
35
3
38
5

We analyze this question by saying, as there are 5 fifths in one quart, there will be 5 times as many fifths as quarts; therefore, in seven quarts and three fifths, there will be 38 fifths, which should be expressed thus, $\frac{38}{5}$. And this fraction, by definition 2d, on page 76, is an improper fraction. Hence the following

RULE.

Multiply the whole number by the denominator of the fraction, and to the product add the numerator, and place their sum over the denominator of the fraction.

7. Reduce $8\frac{3}{11}$ to an improper fraction. Ans. $\frac{91}{11}$.
8. Reduce $15\frac{7}{12}$ to an improper fraction. Ans. $\frac{187}{12}$.
9. In $18\frac{7}{9}$ how many ninths? Ans. $\frac{169}{9}$.
10. In $161\frac{11}{117}$ how many one hundred and seventeenths?
Ans. $\frac{18848}{117}$.
11. Change $43\frac{111}{117}$ to an improper fraction. Ans. $\frac{5142}{117}$.
12. What improper fraction will express $27\frac{2}{13}$?
Ans. $\frac{353}{13}$.
13. Change $111\frac{111}{111}$ to an improper fraction?
Ans. $\frac{12332}{111}$.

IV. To change improper fractions to integers or whole numbers.

MENTAL OPERATIONS.

1. How many dollars in 4 halves? In 5 halves? In 6 halves? In 7 halves? In 12 halves? In 19 halves?
2. How many dollars in 5 quarters? In 9 quarters?
3. How many dollars in 10 eighths? In 20 eighths?

FOR THE SLATE.

4. How many dollars in $\frac{37}{16}$ dollars? Ans. $2\frac{5}{16}$.

OPERATION.

16) 37 ($2\frac{5}{16}$
32
5

This question may be analyzed by saying, as 16 sixteenths make one dollar, there will be as many dollars in 37 sixteenths as 37 contains 16, which is $2\frac{5}{16}$ times, $= \$2\frac{5}{16}$. This answer is called a *mixed number* by definition 5th, page 77. Hence we see the propriety of the following

RULE.

Divide the numerator by the denominator, and if there be a remainder, place it over the denominator at the right hand of the integer.

5. Change $\frac{178}{17}$ to a mixed number. Ans. $10\frac{8}{17}$.

6. Change $\frac{1111}{11}$ to a mixed number. Ans. $10\frac{1}{11}$.
7. Change $\frac{1785}{876}$ to a mixed number. Ans. $1\frac{909}{876}$.
8. Reduce $\frac{1000}{7}$ to a mixed number. Ans. $142\frac{6}{7}$.
9. Reduce $\frac{876}{876}$ to a whole number. Ans. 1.
10. Change $\frac{567}{1}$ to a whole number. Ans. 567.
11. What is the value of $\frac{876}{0}$? Ans.
12. What is the value of $\frac{0}{876}$? Ans.
13. Change 125 to an improper fraction. Ans. $\frac{125}{1}$.

—V. To change or reduce compound fractions to simple fractions.

MENTAL OPERATIONS.

1. What part of an orange is a $\frac{1}{2}$ of a half?
2. What part of an apple is a $\frac{1}{2}$ of a half?
3. What part of a bushel is a $\frac{1}{4}$ of a peck?
4. What part of a quart is a $\frac{1}{2}$ of a pint?

FOR THE SLATE.

5. What is $\frac{4}{5}$ of $\frac{7}{11}$? Ans. $\frac{28}{55}$.

OPERATION.

$\frac{4}{5} \times \frac{7}{11} = \frac{28}{55}$ Ans.

This question may be analyzed by saying, if $\frac{7}{11}$ of an apple be divided into 5 equal parts, that one of these parts is $\frac{1}{55}$ of an apple; and, if $\frac{1}{5}$ of $\frac{7}{11}$ be $\frac{1}{55}$, it is evident, that $\frac{4}{5}$ of $\frac{7}{11}$ will be 7 times as much. 7 times $\frac{1}{55}$ is $\frac{7}{55}$; and, if $\frac{1}{5}$ of $\frac{7}{11}$ be $\frac{7}{55}$, $\frac{4}{5}$ of $\frac{7}{11}$ will be 4 times as much. 4 times $\frac{7}{55}$ is $\frac{28}{55}$.

We therefore induce the following

RULE.

Change mixed numbers and whole numbers, if there be any, to improper fractions; then multiply all the numerators together for a new numerator, and all the denominators together for a new denominator; the fraction should then be reduced to its lowest terms.

6. What is $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{6}{7}$?

OPERATION.

$\frac{2}{3} \times \frac{4}{5} \times \frac{6}{7} = \frac{48}{105} = \frac{16}{35}$ Ans.

7. What is $\frac{7}{8}$ of $\frac{9}{11}$ of 7?

OPERATION.

$$\frac{7}{8} \times \frac{9}{11} \times \frac{7}{1} = \frac{441}{88} = 5\frac{1}{88} \text{ Ans.}$$

8. What is $\frac{7}{8}$ of $\frac{9}{11}$ of $\frac{8}{9}$ of $\frac{7}{7}$? Ans. $\frac{7\cdot8\cdot9}{7\cdot9\cdot8} = \frac{27}{11\cdot8}$.

9. Change $\frac{11}{11}$ of $\frac{1}{2}$ of $\frac{2}{4}$ of $\frac{1}{20}$ of 7 to a simple fraction.

Ans. $\frac{2\cdot1\cdot1}{4\cdot2\cdot20}$.

NOTE 1. If there be numbers in the numerators and denominators, that be alike, an equal number of the same value may be *cancelled*.

10. Reduce $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{7}$ of $\frac{7}{11}$ to a simple fraction.

STATEMENT. CANCELLED.

$$\frac{3\times4\times5\times7}{4\times5\times7\times11} = \frac{3\times\cancel{4}\times\cancel{5}\times\cancel{7}}{\cancel{4}\times\cancel{5}\times\cancel{7}\times11} = \frac{3}{11} \text{ Ans.}$$

In performing this question, we perceive that there is a 4 and 5 and 7 among the numerators, and also the *same* numbers among the denominators; these we *cancel* before we commence the operation.

11. Required the value of $\frac{3}{5}$ of $\frac{4}{11}$ of $\frac{11}{17}$ of $\frac{17}{23}$ of $5\frac{3}{4}$.

STATEMENT.

$$\frac{3\times4\times11\times17\times23}{5\times11\times17\times23\times4} =$$

CANCELLED.

$$\frac{3\times\cancel{4}\times\cancel{11}\times\cancel{17}\times\cancel{23}}{5\times\cancel{11}\times\cancel{17}\times\cancel{23}\times\cancel{4}} = \frac{3}{5} \text{ Ans.}$$

12. Reduce $\frac{1}{7}$ of $\frac{8}{9}$ of $\frac{9}{11}$ of $\frac{5}{8}$ of $\frac{3}{7}$ to a simple fraction.

STATEMENT. CANCELLED.

$$\frac{1\times8\times9\times5\times3}{5\times9\times11\times8\times7} = \frac{1\times\cancel{8}\times\cancel{9}\times\cancel{5}\times3}{\cancel{5}\times\cancel{9}\times11\times\cancel{8}\times7} = \frac{3}{77} \text{ Ans.}$$

13. Reduce $\frac{3}{7}$ of $\frac{4}{11}$ of $\frac{7}{8}$ of $\frac{8}{10}$ of $4\frac{1}{3}$ to a simple fraction.

Ans. $\frac{28}{55}$.

NOTE 2. When there are any two numbers, one in the numerators and the other in the denominators, which may be divided by a number without a remainder, the quotients arising from such division may be used in the operation of the question instead of the original numbers.

14. Reduce $1\frac{1}{3}$ of $\frac{3}{5}$ of $\frac{7}{11}$ to a simple fraction.

STATEMENT. CANCELLED.

$$\frac{15\times 8\times 7}{16\times 9\times 11}=\frac{\overset{5}{\cancel{15}}\times\overset{}{\cancel{8}}\times 7}{\underset{2}{\cancel{16}}\times\underset{3}{\cancel{9}}\times 11}=\frac{35}{66}\text{ Ans.}$$

In performing this question, we find that the 15 among the numerators and the 9 among the denominators may be divided by 3, and that the quotients will be 5 and 3. We write the 5 *above* the 15, and the 3 *below* the 9. We also find an 8 among the numerators, and a 16 among the denominators, which may be divided by 8, and that the quotients will be 1 and 2. We write the 1 *over* the 8, and the 2 *under* the 16. We then multiply the 5, and 1, and 7 together for a new numerator, and the 2, and 3, and 11 together for a new denominator. That the result will be the same by this process as by the other, is evident from the fact, that the multiples of any number have the same ratio to each other, as the numbers themselves.

This cancelling principle, when well understood, will often facilitate the operations of many questions, when the divisors and dividends have a *common denominator*.

15. Reduce $\frac{8}{11}$ of $\frac{22}{35}$ of $\frac{15}{22}$ of $9\frac{1}{8}$ to a whole number.

STATEMENT. CANCELLED.

$$\frac{8\times 22\times 15\times 77}{11\times 35\times 22\times 8}=\frac{\overset{1}{\cancel{8}}\times\overset{}{\cancel{22}}\times\overset{3}{\cancel{15}}\times\overset{11}{\cancel{77}}}{\underset{}{\cancel{11}}\times\underset{5}{\cancel{35}}\times\underset{}{\cancel{22}}\times\underset{1}{\cancel{8}}}=\frac{3}{1}=3\text{ Ans.}$$

16. Divide the continued product of 18, 24, 27, and 30, by the continued product of 20, 21, 9, and 10.

STATEMENT. CANCELLED.

$$\frac{18\times 24\times 27\times 30}{20\times 21\times 9\times 10}=\frac{\overset{2}{\cancel{18}}\times\overset{6}{\cancel{24}}\times\overset{9}{\cancel{27}}\times\overset{3}{\cancel{30}}}{\underset{5}{\cancel{20}}\times\underset{7}{\cancel{21}}\times\underset{1}{\cancel{9}}\times\underset{1}{\cancel{10}}}=\frac{324}{35}=9\tfrac{9}{35}\text{ Ans.}$$

17. Divide the continued product of 20, 19, 18, 17, 16, 15, 14, 13, 12, and 11, by the continued product of 10, 9, 8, 7, 6, 5, 4, 3, 2, and 1.

CANCELLED.

$$\frac{\cancel{2}\cancel{0}\times 19\times\cancel{1}\cancel{8}\times 17\times\cancel{1}\cancel{6}\times\cancel{1}\cancel{5}\times\cancel{1}\cancel{4}\times 13\times\cancel{1}\cancel{2}\times 11}{\cancel{1}\cancel{0}\times\cancel{9}\times\cancel{8}\times\cancel{7}\times\cancel{6}\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1}} = \frac{2\ 2\ 2\ 2\ 2}{1\ 1\ 1\ 1\ 1\ 1} = 184756 \text{ Ans.}$$

NOTE. In this question the product of the quotients of 2, 3, 2, and 2 is cancelled by the product of 4, 3, and 2 in the lower line. Any numbers may be cancelled, when their product is equal to the product of certain other numbers, as in the following question.

18. Divide the continued product of 4, 9, 3, 8, and 225 by the continued product 6, 6, 4, 6, and 11.

STATEMENT. CANCELLED.

$$\frac{4\times 9\times 3\times 8\times 225}{6\times 6\times 4\times 6\times 11} = \frac{\cancel{4}\times\cancel{9}\times\cancel{3}\times\cancel{8}\times 225}{\cancel{6}\times\cancel{6}\times\cancel{4}\times\cancel{6}\times 11} = \frac{225}{11} = 20\tfrac{5}{11} \text{ Ans.}$$

As the product of 4 times 9 in the upper line is equal to the product of 6 times 6 in the under line, they cancel each other; and as the product of 3 times 8 in the upper line is equal to 4 times 6 in the under line, they cancel each other.

VI. To find the least common multiple of two or more numbers, that is, to find the least number, that may be divided by them without a remainder.

RULE.

Divide by such a number, as will divide most of the given numbers without a remainder, and set the several quotients with the several undivided numbers in a line beneath, and so continue to divide, until no number, greater than unity, will divide two or more of them. Then multiply all the divisors, quotients, and undivided numbers together, and the product is the least common multiple.

1. What is the least common multiple of 8, 4, 3, 6?

2)	8	4	3	6
2)	4	2	3	3
3)	2	1	3	3
	2	1	1	1

It is evident, that 24 is a composite number, and that it is composed of the factors 2, 2, 3, and 2; and, therefore, it may be divided by any number, which is the

$2\times 2\times 3\times 2 = 24$ Ans.

product of any two of them; and, as the given numbers are either some one of these, or such a number as may be produced by the product of two or more of them, it is evident, therefore, that 24 may be divided by either of them without a remainder. Q. e. d.

2. What is the least common multiple of 7, 14, 21, and 15? Ans. 210.

3. What is the least common multiple of 3, 4, 5, 6, 7, and 8? Ans. 840.

4. What is the least number, that 10, 12, 16, 20, and 24 will divide without a remainder? Ans. 240.

5. Five men start from the same place to go round a certain island. The first can go round it in 10 days; the second in 12 days; the third in 16 days; the fourth in 18 days; the fifth in 20 days. In what time will they all meet at the place from which they started? Ans. 720 days.

VII. To reduce fractions to a common denominator; that is, to change fractions to other fractions, all having their denominators *alike*, yet retaining the same value.

1. Reduce $\frac{3}{4}$, $\frac{4}{6}$, and $\frac{7}{8}$ to a common denominator.

First Method.

OPERATION.

4) 4 6 8	4×2×3=	24	common denominator.
2) 1 6 2	4	6×3=18	numerator for $\frac{3}{4}=\frac{18}{24}$.
1 3 1	6	4×5=20	numerator for $\frac{4}{6}=\frac{20}{24}$.
	8	3×7=21	numerator for $\frac{7}{8}=\frac{21}{24}$.

Having first obtained a common multiple of all the denominators of the given fractions by the last rule, we assume this, as the common denominator required. This number (24) we divide by the denominators of the given fractions, 4, 6, and 8, and find their quotients to be 6, 4, and 3, which we place under the 24; these numbers we multiply by the numerators, 3, 5, and 7, and find their products to be 18, 20, and 21, and these numbers are the numerators of the fractions required.

Second Method.

OPERATION.

$3 \times 6 \times 8 = 144$ numerator for $\frac{3}{4} = \frac{144}{192}$.
$5 \times 4 \times 8 = 160$ numerator for $\frac{5}{6} = \frac{160}{192}$.
$7 \times 4 \times 6 = 168$ numerator for $\frac{7}{8} = \frac{168}{192}$.
$4 \times 6 \times 8 = 192$ common denominator.

Note. It will be perceived, that this method does not express the fractions in so low terms as the other.

From the above illustration we deduce the following

RULE.

Let compound fractions be reduced to simple fractions, mixed numbers to improper fractions, and whole numbers to improper fractions, by writing a unit under them; then find the least common multiple of all the denominators by the last rule, and it will be the denominator required. Divide the common multiple by each of the denominators, and multiply the quotients by the respective numerators of the fractions, and their products will be the numerators required.

Or, multiply each numerator into all the denominators except its own for a new numerator; and all the denominators into each other for a common denominator.

2. Reduce $\frac{3}{4}$ and $\frac{5}{6}$ to a common denominator.
Ans. $\frac{9}{12}$, $\frac{10}{12}$.

3. Reduce $\frac{7}{9}$, $\frac{4}{15}$, and $\frac{11}{20}$. Ans. $\frac{140}{180}$, $\frac{48}{180}$, $\frac{99}{180}$.

4. Reduce $\frac{4}{7}$, $\frac{3}{14}$, and $\frac{5}{21}$. Ans. $\frac{24}{42}$, $\frac{9}{42}$, $\frac{10}{42}$.

5. Reduce $\frac{8}{15}$, $\frac{5}{35}$, and $\frac{1}{2}$. Ans. $\frac{16}{30}$, $\frac{5}{30}$, $\frac{15}{30}$.

6. Change $\frac{1}{6}$, $\frac{5}{12}$, $\frac{8}{9}$, and $\frac{7}{15}$. Ans. $\frac{30}{180}$, $\frac{75}{180}$, $\frac{160}{180}$, $\frac{84}{180}$.

7. Change $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{7}{8}$. Ans. $\frac{90}{120}$, $\frac{96}{120}$, $\frac{100}{120}$, $\frac{105}{120}$.

8. Change $\frac{3}{4}$, $\frac{2}{5}$, $\frac{4}{9}$, and $\frac{3}{11}$. Ans. $\frac{1485}{1980}$, $\frac{792}{1980}$, $\frac{880}{1980}$, $\frac{360}{1980}$.

9. Reduce $\frac{7}{8}$, $\frac{9}{10}$, and $7\frac{3}{4}$. Ans. $\frac{35}{40}$, $\frac{36}{40}$, $\frac{310}{40}$.

10. Reduce $\frac{3}{7}$, $\frac{9}{14}$, $\frac{11}{28}$, and $5\frac{3}{7}$. Ans. $\frac{12}{28}$, $\frac{18}{28}$, $\frac{11}{28}$, $\frac{152}{28}$.

11. Reduce $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{5}{12}$. Ans. $\frac{12}{24}$, $\frac{18}{24}$, $\frac{20}{24}$, $\frac{15}{24}$, $\frac{21}{24}$, $\frac{10}{24}$.

12. Change $\frac{4}{9}$, $\frac{2}{3}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{12}$. Ans. $\frac{16}{36}$, $\frac{24}{36}$, $\frac{12}{36}$, $\frac{9}{36}$, $\frac{6}{36}$, $\frac{3}{36}$.

13. Reduce $\frac{5}{6}$, $\frac{4}{9}$, and $\frac{7}{12}$. Ans. $\frac{30}{36}$, $\frac{16}{36}$, $\frac{21}{36}$.

14. Change $7\frac{3}{4}$, $5\frac{6}{11}$, 7, and 8. Ans. $\frac{341}{44}$, $\frac{244}{44}$, $\frac{308}{44}$, $\frac{352}{44}$.

15. Change $\frac{3}{4}$, 4, 5, 7, and 9. Ans. $\frac{3}{4}$, $\frac{16}{4}$, $\frac{20}{4}$, $\frac{28}{4}$, $\frac{36}{4}$.

VIII. To reduce fractions of a lower denomination to a higher.

1. Reduce $\frac{1}{3}$ of a farthing to the fraction of a pound.

OPERATION.

$\frac{1}{4} \times \frac{4}{3}$qr. $= \frac{4}{36} = \frac{1}{9}$d.
$\frac{1}{12} \times \frac{1}{9}$d. $= \frac{1}{108}$s.
$\frac{1}{20} \times \frac{1}{108}$s. $= \frac{1}{2160}$£.

$\frac{1}{2160}$ Ans.

This question may be analyzed thus; since 4 farthings make a penny, there will be $\frac{1}{4}$ as many pence as farthings; therefore $\frac{1}{4}$ of $\frac{1}{3}$ of a farthing is $\frac{1}{36} = \frac{1}{9}$ of a penny. Again, as 12 pence make a shilling, there will be $\frac{1}{12}$ as many shillings as pence, therefore $\frac{1}{12}$ of $\frac{1}{9}$ of a penny is $\frac{1}{108}$ of a shilling. As 20 shillings make a pound, there will be $\frac{1}{20}$ as many pounds as shillings, therefore $\frac{1}{20}$ of $\frac{1}{108}$ of a shilling is $\frac{1}{2160}$ of a pound. Q. e. d.

The operation of this question may be abridged thus:

OPERATION.

$$\frac{1}{9} \times \frac{1}{4} \times \frac{1}{12} \times \frac{1}{20} = \frac{1}{2160} \text{ Ans.}$$

Hence the following

RULE.

Let the given fraction be reduced to a compound one by comparing it with all the denominations between the given one and the one to which it is required to reduce it; then reduce this compound fraction to a simple one.

2. Reduce $\frac{1}{7}$ of a grain Troy to the fraction of a pound.

$$\frac{4 \times 1 \times 1 \times 1}{7 \times 24 \times 20 \times 12} = \frac{1}{10080} \text{ Ans.}$$

3. What part of an ounce is $\frac{3}{10}$ of a scruple?

$$\frac{3 \times 1 \times 1}{10 \times 3 \times 8} = \frac{1}{80} \text{ Ans.}$$

4. What part of a ton is $\frac{1}{5}$ of an ounce?

$$\frac{1 \times 1 \times 1 \times 1 \times 1}{5 \times 16 \times 28 \times 1 \times 20} = \frac{1}{44800} \text{ Ans.}$$

5. What part of a mile is $\frac{8}{9}$ of a rod?

$$\frac{8 \times 1 \times 1}{9 \times 40 \times 8} = \frac{1}{360} \text{ Ans.}$$

6. What part of 3 acres is $\frac{4}{9}$ of a square foot?

$$\frac{4 \times 1 \times 1 \times 1 \times 1}{9 \times 272\frac{1}{4} \times 40 \times 4 \times 3} = \frac{1}{294030} \text{ Ans.}$$

7. What part of 3hhds. is $\frac{4}{7}$ of a quart?

$$\frac{4 \times 1 \times 1 \times 1}{7 \times 4 \times 63 \times 3} = \frac{1}{1323} \text{ Ans.}$$

8. What part of 3 yards square, are 3 square yards?

Ans. $\frac{1}{3}$.

9. What part of $\frac{1}{8}$ of a solid foot is $\frac{1}{8}$ of a yard solid?

Ans. $\frac{1}{4}$.

IX. To reduce fractions of a higher denomination to a lower.

1. Reduce $\frac{1}{1400}$ of a pound to the fraction of a farthing.

Ans. $\frac{24}{35}$.

We explain this question in the following manner.

OPERATION.

$\frac{1}{1400} \times \frac{20}{1} = \frac{20}{1400} = \frac{1}{70}$s.
$\frac{1}{70} \times \frac{12}{1} = \frac{12}{70} = \frac{6}{35}$d.
$\frac{6}{35} \times \frac{4}{1} = \frac{24}{35}$qr. Ans.

As shillings are twentieths of a pound, there will be 20 times as many parts of a shilling in $\frac{1}{1400}$ of a pound, as there are parts of a pound; therefore $\frac{1}{1400}$ of a pound is equal to $\frac{1}{1400}$ of $\frac{20}{1} = \frac{20}{1400} = \frac{1}{70}$ of a shilling. And as pence are twelfths of shillings, there will be twelve times as many parts of a penny in $\frac{1}{70}$ of a shilling, as there are parts of a shilling; therefore $\frac{1}{70}$ of a shilling is equal to $\frac{1}{70}$ of $\frac{12}{1} = \frac{12}{70} = \frac{6}{35}$ of a penny. Again, as farthings are fourths of a penny, there will be 4 times as many parts of a farthing in $\frac{6}{35}$ of a penny, as there are parts of a penny; therefore $\frac{6}{35}$ of a penny are equal to $\frac{6}{35}$ of $\frac{4}{1} = \frac{24}{35}$ of a farthing. Q. e. d.

The operation of this question may be facilitated by the following manner.

OPERATION.

$$\frac{1}{1400} \times \frac{20}{1} \times \frac{12}{1} \times \frac{4}{1} = \frac{960}{1400} = \frac{24}{35}\text{qr. Ans.}$$

Hence the following

RULE.

Let the given numerator be multiplied by all the denominations between it and the one to which it is to be reduced; then place the product over this denominator, and reduce the fraction to its lowest terms.

2. What part of a grain is $\frac{1}{5640}$ of a pound Troy?

$$\frac{1}{5640} \times \frac{12}{1} \times \frac{20}{1} \times \frac{24}{1} = \frac{5760}{5640} = \frac{24}{23} \text{ Ans.}$$

3. Reduce $\frac{1}{1320}$ of a furlong to the fraction of a foot.

$$\frac{1}{1320} \times \frac{40}{1} \times \frac{16\frac{1}{2}}{1} = \frac{660}{1320} = \frac{1}{2} \text{ Ans.}$$

4. What part of a square foot is $\frac{1}{58080}$ of an acre?

$$\frac{1}{58080} \times \frac{4}{1} \times \frac{40}{1} \times \frac{272\frac{1}{4}}{1} = \frac{43560}{58080} = \frac{3}{4} \text{ Ans.}$$

5. What part of a peck is $\frac{3}{14}$ of a bushel? Ans. $\frac{3}{7}$.
6. What part of a pound is $\frac{1}{200}$ of a cwt.? Ans. $\frac{14}{25}$.

X. To find the value of a fraction in the known parts of the integer.

RULE.

Multiply the numerator by the next lower denomination of the integer, and divide the product by the denominator; if any thing remains, multiply it by the next less denomination, and divide as before, and so continue, as far as may be required; and the several quotients will be the answer.

1. What is the value of $\frac{7}{24}$ of a pound? Ans. 5s. 10d.

OPERATION.

£	s.	d.
1	0	0
		7
24)7	0	0
0	5	10

2. What is the value of $\frac{7}{8}$ af a cwt.?
Ans. 3qr. 3lb. 1oz. $12\frac{4}{8}$dr.

OPERATION.

Cwt.	qr.	lb.	oz.	dr.
1	0	0	0	0
				7
9)7	0	0	0	0
0	3	3	1	$12\frac{4}{9}$

3. What is the value of $\frac{7}{9}$ of a yard? Ans. 3qr. $0\frac{4}{9}$na.

OPERATION.

Yd.	qr.	na.
1	0	0
		7
9)7	0	0
0	3	$0\frac{4}{9}$

4. What is the value of $\frac{3}{7}$ of an acre?
Ans. 1R. 28p. 155ft. $82\frac{2}{7}$in.

OPERATION.

A	R.	p.	ft.	in.
1	0	0	0	0
				3
7)3	0	0	0	0
0	1	28	155	$82\frac{2}{7}$

5. What is the value of $\frac{2}{3}$ of a mile?
Ans. 1fur. 31rd. 1ft. 10in.

OPERATION.

M.	fur.	rd.	ft.	in.
1	0	0	0	0
				2
9)2	0	0	0	0
0	1	31	1	10

6. What is the value of $\frac{8}{11}$ of an ell English?
Ans. 1qr. $1\frac{5}{11}$na.

OPERATION.

EE.	qr.	na.
1	0	0
		3
11)3	0	0
0	1	$1\frac{5}{11}$

7. What is the value of $\frac{3}{4}$ of a hogshead of wine?
Ans. 18gal. 0qt. 0pt.

8. What is the value of $\frac{7}{11}$ of a year?
Ans. 232da. 10h. 21m. $49\frac{1}{11}$sec.

XI. To reduce any mixed quantity of weights, measures, &c. to the fractions of the integer.

1. What part of a pound is 3s. 6d.?

OPERATION.

3s. 6d. = 42d.
20s. = 240d. = $\frac{7}{40}$ Ans.

To perform this question, we reduce the 3s. 6d. to pence, it being the lowest denomination in the question, and we make them the numerator of the fraction. We then reduce the one pound to pence, and make them the denominator of the fraction. This fraction we reduce to its lowest terms, and we have the answer required; wherefore the following

RULE.

Reduce the given number to the lowest denomination it contains for a numerator, and reduce the integers to the same denomination, for the denominator of the fraction required.

2. Reduce 4s. 8d. to the fraction of a pound.

OPERATION.

4s. 8d. = 56d.
20s. = 240d. = $\frac{7}{30}$ Ans.

3. What part of a ton is 4cwt. 3qr. 12lb.?

OPERATION.

4cwt. 3qr. 12lb. = 544lb.
20cwt. = 2240lb. = $\frac{17}{70}$ Ans.

4. What part of 2m. 3fur. 20rd. is 2fur. 30rd.?

OPERATION.

2fur. 30rd. = 110rd.
2m. 3fur. 20rd. = 780rd. = $\frac{11}{78}$ Ans.

5. What part of 2A. 2R. 32p. is 3R. 24p.?

OPERATION.

$$\begin{array}{r} 3\text{R. } 24\text{p.} = 144\text{p.} \\ 2\text{A. } 2\text{R. } 32\text{p.} = 432\text{p.} \end{array} = \tfrac{1}{3} \text{ Ans.}$$

6. What part of a hogshead of wine is 18gal. 2qt. ?
Ans. $\frac{37}{126}$.

7. What part of 30 days are 8 days 17h. 20m. ?
Ans. $\frac{157}{510}$.

8. From a piece of cloth, containing 13yd. 0qr. 2na. there were taken 5yd. 2qr. 2na. What part of the whole piece was taken ? Ans. $\frac{3}{7}$.

Section 21.

ADDITION OF VULGAR FRACTIONS.

I. To add fractions, that have a common denominator.

RULE.

Write the sum of the numerators over the common denominator.

1. Add $\frac{1}{11}$, $\frac{2}{11}$, $\frac{4}{11}$, $\frac{5}{11}$, and $\frac{6}{11}$ together.

OPERATION.

$$1 + 2 + 4 + 5 + 6 = \tfrac{18} = 2\tfrac{4}{7} \text{ Ans.}$$

2. Add $\frac{4}{11}$, $\frac{5}{11}$, $\frac{7}{11}$, $\frac{8}{11}$, $\frac{9}{11}$, and $\frac{10}{11}$ together. Ans. $3\frac{10}{11}$.
3. Add $\frac{4}{17}$, $\frac{3}{17}$, $\frac{8}{17}$, $\frac{9}{17}$, and $\frac{11}{17}$ together. Ans. $2\frac{1}{17}$.
4. Add $\frac{3}{25}$, $\frac{8}{25}$, $\frac{18}{25}$, and $\frac{21}{25}$ together. Ans. $2\frac{1}{25}$.
5. Add $\frac{11}{17}$, $\frac{13}{17}$, $\frac{3}{17}$, and $\frac{4}{17}$ together. Ans. $2\frac{13}{17}$.
6. Add $\frac{113}{137}$, $\frac{111}{137}$, and $\frac{19}{137}$ together. Ans. $1\frac{113}{137}$.
7. Add $\frac{1678}{1771}$, $\frac{1134}{1771}$, and $\frac{198}{1771}$ together. Ans. $1\frac{1906}{1771}$.

II. To add fractions that have not a common denominator.

RULE.

Reduce mixed numbers to improper fractions, and compound fractions to simple fractions; then reduce all the

fractions to a common denominator; and the sum of their numerators, written over the common denominator, will be the answer required.

1. What is the sum of $\frac{5}{6}$, $\frac{3}{8}$, and $\frac{7}{12}$?

OPERATION.

2)	6	8	12	$2\times3\times2\times2=24$	common denominator
3)	3	4	6	6	$4\times5=20$
2)	1	4	2	8	$3\times3=9$
	1	2	1	12	$2\times7=14$
					43
					24

$= 1\frac{19}{24}$ Ans.

2. What is the sum of $\frac{5}{6}$, $\frac{11}{12}$, and $\frac{13}{16}$? Ans. $2\frac{17}{48}$.

3. What is the sum of $\frac{9}{20}$, $\frac{11}{15}$, and $\frac{5}{12}$? Ans. $1\frac{58}{120}$.

4. What is the sum of $\frac{19}{21}$, and $\frac{31}{37}$? Ans. $1\frac{377}{777}$.

5. What is the sum of $\frac{3}{4}$, $\frac{5}{6}$, $\frac{8}{9}$, and $\frac{1}{12}$? Ans. $2\frac{1}{24}$.

6. Add $\frac{4}{5}$, $\frac{9}{21}$, $\frac{11}{24}$, and $\frac{1}{2}$ together. Ans. $1\frac{233}{504}$.

7. Add $\frac{19}{72}$, $\frac{5}{84}$, and $\frac{71}{56}$ together. Ans. $1\frac{233}{2016}$.

8. Add $\frac{3}{25}$, $\frac{19}{50}$, $\frac{74}{75}$, and $\frac{81}{100}$ together. Ans. $2\frac{268}{300}$.

9. Add $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, and $\frac{7}{8}$ together. Ans. $5\frac{79}{280}$.

10. Add $\frac{8}{9}$, $\frac{9}{10}$, $\frac{10}{11}$, $\frac{11}{12}$, $\frac{12}{13}$, $\frac{13}{14}$, and $\frac{14}{15}$ together.

Ans. $6\frac{1440}{3603}$.

11. Add $\frac{3}{4}$ of $\frac{2}{3}$ to $\frac{5}{6}$ of $\frac{7}{8}$. Ans. $1\frac{11}{48}$.

12. Add $\frac{3}{4}$ of $\frac{7}{8}$ to $\frac{11}{12}$ of $\frac{1}{2}$. Ans. $1\frac{11}{96}$.

13. Add $\frac{1}{5}$ of $\frac{3}{8}$ to $\frac{1}{5}$ of $\frac{7}{10}$. Ans. $\frac{238}{1350}$.

14. Add $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ to $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{7}{10}$. Ans. $\frac{8}{10}$.

15. Add $\frac{1}{2}$ of $\frac{3}{11}$ of $1\frac{1}{2}$ to $\frac{1}{2}$ of $\frac{3}{5}$. Ans. $\frac{7}{55}$.

16. Add $3\frac{3}{7}$ to $4\frac{11}{14}$. Ans. $8\frac{3}{14}$.

17. Add $4\frac{3}{4}$ to $5\frac{4}{7}$. Ans. $10\frac{17}{28}$.

18. Add $17\frac{3}{4}$ to $18\frac{5}{12}$. Ans. $36\frac{1}{6}$.

NOTE 1. If the quantities are mixed numbers, the better way is to add the fractional parts separately, and then to add their sum to the amount of the whole numbers.

NOTE 2. If there be but two fractions to add, and their numerators are a unit, their sum may be found by writing the sum of the

denominators over their product; thus, if it were required to find the sum of $\frac{1}{3}$ and $\frac{1}{7}$, we should add the 3 and 7 together for a numerator, and multiply them together for a denominator, and the fraction would be $\frac{10}{21}$.

19. Add $\frac{1}{4}$ to $\frac{1}{5}$, $\frac{1}{7}$ to $\frac{1}{8}$, $\frac{1}{2}$ to $\frac{1}{3}$, $\frac{1}{5}$ to $\frac{1}{6}$, $\frac{1}{2}$ to $\frac{1}{5}$.

20. Add $\frac{1}{2}$ to $\frac{1}{11}$, $\frac{1}{8}$ to $\frac{1}{9}$, $\frac{1}{7}$ to $\frac{1}{8}$, $\frac{1}{5}$ to $\frac{1}{12}$, $\frac{1}{3}$ to $\frac{1}{10}$, $\frac{1}{4}$ to $\frac{1}{7}$.

21. Add $\frac{1}{5}$ to $\frac{1}{7}$, $\frac{1}{7}$ to $\frac{1}{12}$, $\frac{1}{6}$ to $\frac{1}{8}$, $\frac{1}{6}$ to $\frac{1}{12}$, $\frac{1}{3}$ to $\frac{1}{5}$, $\frac{1}{11}$ to $\frac{1}{12}$.

Section 22.

SUBTRACTION OF VULGAR FRACTIONS.

I. To subtract fractions, that have a common denominator.

RULE.

Subtract the less numerator from the greater, and under the remainder write the common denominator, and reduce the fraction if necessary.

OPERATION.

1. From $\frac{7}{9}$ take $\frac{2}{9}$. $7-2=5$, $\frac{5}{9}$ Ans.
2. From $\frac{7}{11}$ take $\frac{2}{11}$. Ans. $\frac{5}{11}$.
3. From $\frac{11}{19}$ take $\frac{7}{19}$. Ans. $\frac{4}{19}$.
4. From $\frac{27}{37}$ take $\frac{4}{37}$. Ans. $\frac{23}{37}$.
5. From $\frac{161}{711}$ take $\frac{19}{711}$. Ans. $\frac{142}{711}$.
6. From $\frac{6258}{17258}$ take $\frac{1150}{17258}$. Ans. $\frac{2554}{8629}$.
7. From $\frac{7}{20}$ take $\frac{5}{20}$. Ans. $\frac{1}{10}$.
8. From $\frac{97}{100}$ take $\frac{17}{100}$. Ans. $\frac{4}{5}$.

II. To subtract fractions whose denominators are unlike.

RULE.

Reduce the fractions to a common denominator, as in Addition of fractions; then write the difference of the numerators over the common denominator.

9. From $\frac{13}{16}$ take $\frac{7}{12}$. Ans. $\frac{11}{48}$.

OPERATION.

$$\begin{array}{r|l} 4)16 \quad 12 & 4\times4\times3=48 \text{ common denominator.} \\ \hline 4 \quad 3 & \\ & \begin{array}{r|l} 16 & 3\times13=39 \\ 12 & 4\times\ 7=28 \\ \hline & 11 \end{array} \\ & \overline{48} \text{ Ans.} \end{array}$$

10. From $9\frac{7}{8}$ take $5\frac{11}{12}$. Ans. $3\frac{23}{24}$

OPERATION.

$$9\tfrac{7}{8}=\tfrac{79}{8},\ 5\tfrac{11}{12}=\tfrac{71}{12}.$$

$$\begin{array}{r|l} 4)8 \quad 12 & 4\times2\times3=24 \text{ common denominator.} \\ \hline 2 \quad 3 & \\ & \begin{array}{r|l} 8 & 3\times79=237 \\ 12 & 2\times71=142 \\ \hline & 95 \end{array} \\ & \frac{95}{24}=3\frac{23}{24} \text{ Ans.} \end{array}$$

11. From $\frac{3}{8}$ of $12\frac{1}{6}$ take $\frac{2}{5}$ of $9\frac{7}{12}$. Ans. $\frac{17}{48}$.

OPERATION.

$$12\tfrac{1}{6}=\tfrac{73}{6},\ 9\tfrac{7}{12}=\tfrac{115}{12}$$

$$\tfrac{3}{8}\times\tfrac{73}{6}=\tfrac{219}{48},\ \tfrac{2}{5}\times\tfrac{115}{12}=\tfrac{230}{60}=\tfrac{23}{6}$$

$$\tfrac{219}{48}-\tfrac{23}{6},\ 6)\begin{array}{r|l} 48 & 6 \\ \hline 8 & 1 \end{array}$$

$$6\times8\times1=48 \text{ common denominator.}$$

$$\begin{array}{r|l} 48 & 1\times231=231 \\ 6 & 8\times\ 23=184 \\ \hline & 47 \\ & \overline{48} \text{ Ans.} \end{array}$$

12. From $\frac{7}{15}$ take $\frac{4}{21}$. Ans. $\frac{23}{105}$.

13. From $\frac{13}{20}$ take $\frac{11}{16}$. Ans. $\frac{21}{80}$.

14. From $\frac{17}{24}$ take $\frac{7}{20}$. Ans. $\frac{43}{120}$.

15. From $\frac{11}{34}$ take $\frac{1}{10}$. Ans. $\frac{12}{85}$.

16. From $\frac{3}{16}$ take $\frac{3}{16}$. Ans. $\frac{43}{144}$.

17. From $\frac{18}{37}$ take $\frac{3}{11}$. Ans. $\frac{87}{407}$.

18. From $\frac{111}{200}$ take $\frac{1}{19}$. Ans. $\frac{1909}{3800}$.

19. From $\frac{1}{10}$ take $\frac{1}{1000}$. Ans. $\frac{99}{1000}$.

20. From $\frac{3}{8}$ of $\frac{9}{11}$ take $\frac{1}{4}$ of $\frac{3}{7}$. Ans. $\frac{123}{154}$.

21. From $\frac{1}{6}$ of $\frac{9}{10}$ take $\frac{1}{12}$ of $1\frac{3}{5}$. Ans. $\frac{3}{120}$.

22. From $7\frac{1}{2}$ take $3\frac{7}{9}$. Ans. $3\frac{17}{18}$.

23. From $8\frac{3}{7}$ take $5\frac{4}{5}$. Ans. $2\frac{23}{35}$.

24. From $9\frac{1}{4}$ take $3\frac{7}{9}$. Ans. $5\frac{8}{36}$.

25. From $10\frac{3}{4}$ take $10\frac{1}{19}$. Ans. $\frac{53}{76}$.

III. To subtract a proper or mixed fraction from a whole number.

26. From 16 take $1\frac{1}{4}$. Ans. $14\frac{3}{4}$.

OPERATION.

From 16
Take $1\frac{1}{4}$
$14\frac{3}{4}$

To subtract the $\frac{1}{4}$ in this example, 1 must be borrowed from the 6 in the minuend, and reduced to fourths, ($\frac{4}{4}$), and the $\frac{1}{4}$ must be taken from them; $\frac{1}{4}$ from $\frac{4}{4}$ leaves $\frac{3}{4}$. To pay for the 1, which was borrowed, 1 must be added to the 1 in the subtrahend, $1+1=2$; and 2 taken from 16 leaves 14, and the $\frac{3}{4}$, placed at the right hand of it, gives the answer $14\frac{3}{4}$. The same result will be obtained, if we adopt the following

RULE.

Subtract the numerator from the denominator of the fraction, and under the remainder write the denominator, and carry one to the subtrahend to be subtracted from the minuend.

OPERATION.

	27.	28.	29.	30.	31.
From	16	19	13	14	17
Take	$1\frac{1}{4}$	$3\frac{3}{7}$	$9\frac{1}{11}$	$8\frac{3}{7}$	$6\frac{11}{12}$
	$14\frac{3}{4}$	$15\frac{4}{7}$	$3\frac{10}{11}$	$5\frac{4}{7}$	$10\frac{1}{12}$

If it be required to subtract one mixed number from another mixed number, the following method may be adopted.

32. From $9\frac{2}{7}$ take $3\frac{3}{5}$. Ans. $5\frac{24}{35}$.

OPERATION.

Minuend $9\frac{2}{7} = 9\frac{10}{35}$
Subtrahend $3\frac{3}{5} = 3\frac{21}{35}$
$5\frac{24}{35}$ Ans.

In this question, we multiply the 2 and the 7, the numerator and denominator of the fraction in the minuend by 5, the

denominator of the fraction in the subtrahend, and we have a new fraction $\frac{10}{35}$, which we write at the right hand of the other 9, thus, $9\frac{10}{35}$. We then multiply the numerator and denominator of the subtrahend by 7, the denominator of the minuend, and we have another new fraction, $\frac{21}{35}$, which we place at the right hand of the other 3, thus, $3\frac{21}{35}$. It will now be perceived, that we have changed the fractions $9\frac{2}{7}$ and $3\frac{3}{5}$ to other fractions of the same value, having a common denominator. We now subtract as in question 26th by adding 1 ($\frac{35}{35}$) to $\frac{10}{35}$, which makes $\frac{45}{35}$, and from this we subtract $\frac{21}{35}$; thus, $\frac{45}{35} - \frac{21}{35} = \frac{24}{35}$, we then carry the 1 we borrowed to the 3, $1 + 3 = 4$, which we take from 9, and find 5 remaining. The answer therefore is $5\frac{24}{35}$.

	33.	34.	35.	36.	37.
From	$12\frac{2}{7}$	$16\frac{3}{11}$	$19\frac{2}{5}$	$97\frac{1}{4}$	$87\frac{11}{23}$
Take	$9\frac{1}{2}$	$5\frac{6}{7}$	$15\frac{6}{7}$	$18\frac{3}{11}$	$19\frac{3}{4}$
	$2\frac{13}{14}$	$10\frac{36}{77}$	$3\frac{26}{35}$	$78\frac{13}{44}$	$67\frac{67}{92}$

	38.	39.	40.	41.	42.
From	$19\frac{1}{6}$	$15\frac{1}{7}$	$9\frac{1}{13}$	$71\frac{1}{15}$	$61\frac{1}{0}$
Take	$7\frac{3}{11}$	$8\frac{1}{4}$	$3\frac{18}{13}$	$13\frac{7}{12}$	$15\frac{0}{11}$

43. From a hhd. of wine there leaked out $12\frac{3}{8}$ gallons, how much remained? Ans. $50\frac{5}{8}$.

44. From \$10, \$$2\frac{1}{2}$ was given to Benjamin, \$$3\frac{1}{4}$ to Lydia, \$$1\frac{1}{2}$ to Emily, and the remainder to Betsey; what did she receive? Ans. \$$3\frac{1}{8}$.

NOTE. If it be required to find the difference between two fractions, whose numerators are a unit, the most ready way will be to write the difference of the denominators over their product.

45. What is the difference between $\frac{1}{3}$ and $\frac{1}{7}$?

OPERATION.

$$7 - 3 = 4$$
$$7 \times 3 = 21, \text{ Ans.}$$

46. Take $\frac{1}{5}$ from $\frac{1}{3}$, $\frac{1}{7}$ from $\frac{1}{2}$, $\frac{1}{5}$ from $\frac{1}{4}$, $\frac{1}{7}$ from $\frac{1}{3}$.

47. Take $\frac{1}{9}$ from $\frac{1}{4}$, $\frac{1}{8}$ from $\frac{1}{7}$, $\frac{1}{4}$ from $\frac{1}{3}$, $\frac{1}{5}$ from $\frac{1}{4}$.

48. Take $\frac{1}{8}$ from $\frac{1}{3}$, $\frac{1}{7}$ from $\frac{1}{2}$, $\frac{1}{12}$ from $\frac{1}{7}$, $\frac{1}{11}$ from $\frac{1}{8}$.

Section 23.

MULTIPLICATION OF VULGAR FRACTIONS.

I. To multiply a fraction by a whole number, or a whole number by a fraction.

Multiply the numerator of the fraction by the whole number, and under the product write the denominator of the fraction.

1. Multiply $\frac{7}{8}$ by 15.

OPERATION.

$\frac{7}{8} \times \frac{15}{1} = \frac{105}{8} = 13\frac{1}{8}$ Ans.

This question may be analyzed as those in compound fractions.

2. Multiply $\frac{11}{12}$ by 83.

OPERATION.

$\frac{11}{12} \times \frac{83}{1} = \frac{913}{12} = 76\frac{1}{12}$ Ans.

3. If a man receive $\frac{3}{8}$ of a dollar for one day's labor, what will he receive for 21 days' labor? Ans. $\$7\frac{7}{8}$.

4. What cost 56lbs. of chalk at $\frac{3}{4}$ of a cent per lb.? Ans. $\$0.42$.

5. What cost 396lbs. of copperas at $\frac{2}{11}$ of a cent per lb.? Ans. $\$3.24$.

6. What cost 79 bushels of salt at $\frac{7}{8}$ of a dollar per bushel? Ans. $\$69\frac{1}{8}$.

7. Multiply 376 by $\frac{11}{17}$. Ans. $243\frac{5}{17}$.

8. Multiply $\frac{15}{17}$ by 189. Ans. $166\frac{13}{17}$.

9. Multiply 471 by $\frac{2}{117}$. Ans. $8\frac{2}{39}$.

10. Multiply 871 by $\frac{1}{37}$. Ans. $23\frac{20}{37}$.

11. Multiply $\frac{113}{117}$ by 365. Ans. $352\frac{61}{117}$.

12. Multiply 867 by $\frac{1}{136}$. Ans. $6\frac{51}{136}$.

II. To multiply a mixed number by a whole number, or a whole number by a mixed number.

13. Multiply $4\frac{3}{5}$ by 7. Ans. $32\frac{1}{5}$.

OPERATION.

$4\frac{3}{5}$
7
$32\frac{1}{5}$ Ans.

In performing this question, we say 7 times 3 fifths are 21 fifths, and 21 fifths are equal to $4\frac{1}{5}$. We write down the $\frac{1}{5}$ and carry the 4 to the product of 7 times $4 = 32$. Hence the following

RULE.

Multiply the numerator of the mixed number by the whole number, and divide the product by the denominator of the fraction; and, as many times as it contains the denominator, so many units must be carried to the product of the integers. If, after division, any thing remains, let it be a numerator, and the divisor a denominator to a fraction to be affixed to the product.

14. Multiply $9\frac{3}{4}$ by 5. Ans. $46\frac{1}{4}$.

15. Multiply $12\frac{2}{5}$ by 7. Ans. $88\frac{1}{5}$.

16. Multiply $8\frac{1}{12}$ by 9. Ans. $80\frac{1}{4}$.

17. Multiply $7\frac{1}{3}$ by 10. Ans. $71\frac{1}{3}$.

18. Multiply $11\frac{8}{9}$ by 8. Ans. $94\frac{8}{9}$.

19. What cost $7\frac{8}{11}$lbs. of beef at 5 cents per pound ? Ans. $37\frac{8}{11}$.

20. What cost $23\frac{7}{12}$bbs. flour at $6 per barrel ? Ans. $141\frac{1}{2}$.

21. What cost $8\frac{3}{4}$yds. cloth at $5 per yard ? Ans. $41\frac{7}{8}$.

22. What cost 9 barrels of vinegar at $6\frac{3}{8}$ per barrel ? Ans. $57\frac{3}{8}$.

23. What cost 12 cords of wood at $6.37\frac{1}{2}$ per cord ? Ans. $76.50.

24. What cost 11cwt. of sugar at $9\frac{3}{8}$ per cwt. ? Ans. $103\frac{1}{8}$.

25. What cost $4\frac{3}{8}$ bushels of rye at $1.75 per bushel ? Ans. $7.65\frac{1}{8}$.

26. What cost 7 tons of hay at $11\frac{7}{8}$ per ton ? Ans. $83\frac{1}{8}$.

27. What cost 9 doz. of adzes at $10\frac{5}{8}$ per doz. ? Ans. $95\frac{5}{8}$.

28. What cost 5 tons of lumber at $3\frac{1}{8}$ per ton ? Ans. $15\frac{5}{8}$.

29. What cost 15cwt. of rice at $7.62\frac{1}{2}$ per cwt. ? Ans. $114.37\frac{1}{2}$.

30. What cost 40 tons coal at $8.37½ per ton?
Ans. $335.00.

III. To multiply simple fractions.

31. Multiply $\frac{7}{8}$ by $\frac{3}{4}$. Ans. $\frac{7}{12}$.

OPERATION.

$\frac{7}{8} \times \frac{3}{4} = \frac{21}{32} = \frac{7}{12}$ Ans.

This question may be analyzed in the same manner as in compound fractions.

Hence the following

RULE.

Multiply the numerators together for a new numerator, and the denominators together for a new denominator; then reduce the fraction to its lowest terms.

32. Multiply $\frac{7}{8}$ by $\frac{8}{11}$. Ans. $\frac{7}{11}$.

OPERATION.

$$\frac{7}{8} \times \frac{8}{11} = \frac{56}{88} = \frac{7}{11} \text{ Ans.}$$

CANCELLED.

$$\frac{7}{\cancel{8}} \times \frac{\cancel{8}}{11} = \frac{7}{11} \text{ Ans.}$$

33. Multiply $\frac{5}{11}$ by $\frac{11}{20}$. Ans. $\frac{1}{4}$.

34. Multiply $\frac{8}{15}$ by $\frac{18}{24}$. Ans. $\frac{1}{5}$.

35. Multiply $\frac{18}{19}$ by $\frac{18}{90}$. Ans. $\frac{1}{5}$.

36. Multiply $\frac{15}{17}$ by $\frac{17}{60}$. Ans. $\frac{1}{4}$.

37. Multiply $\frac{1}{9}$ by $\frac{8}{17}$. Ans. $\frac{8}{153}$.

38. Multiply $\frac{8}{23}$ by $\frac{23}{56}$. Ans. $\frac{1}{7}$.

39. What cost $\frac{7}{8}$ of a bushel of corn at $\frac{8}{9}$ of a dollar per bushel? Ans. $\frac{7}{9}$ of a dollar.

40. If a man travels $\frac{8}{11}$ of a mile in an hour, how far would he travel in $\frac{11}{32}$ of an hour? Ans. $\frac{1}{4}$ of a mile.

41. If a bushel of corn will buy $\frac{7}{10}$ of a bushel of salt, how much salt might be bought for $\frac{3}{4}$ of a bushel of corn? Ans. $\frac{21}{40}$ of a bushel.

NOTE. If there be mixed numbers in the question, they must be reduced to improper fractions, and compound fractions must be reduced to simple fractions.

42. Multiply $4\frac{3}{5}$ by $6\frac{2}{3}$.

OPERATION.

$4\frac{3}{5} = \frac{23}{5}$, $6\frac{2}{3} = \frac{20}{3}$, $\frac{23}{5} \times \frac{20}{3} = \frac{460}{15} = 30\frac{2}{3}$ Ans.

43. Multiply $7\frac{1}{8}$ by $8\frac{3}{7}$. Ans. $60\frac{3}{56}$.

44. Multiply $4\frac{7}{8}$ by $9\frac{1}{4}$. Ans. $45\frac{3}{32}$.

45. Multiply $11\frac{2}{7}$ by $8\frac{4}{5}$. Ans. $99\frac{11}{35}$.

46. Multiply $12\frac{3}{4}$ by $11\frac{5}{8}$. Ans. $147\frac{1}{3}$.

47. What cost $7\frac{3}{4}$ cords of wood at $\$5\frac{3}{8}$ per cord?
Ans. $\$41\frac{21}{32}$.

48. What cost $7\frac{3}{8}$ yds. of cloth at $\$3\frac{1}{2}$ per yard?
Ans. $\$25\frac{13}{16}$.

49. What cost $6\frac{3}{4}$ gallons molasses at $23\frac{3}{4}$ cents per gallon? Ans. $\$152\frac{1}{2}\frac{8}{8}$.

50. If a man travels $3\frac{1}{8}$ miles in one hour, how far will he travel in $9\frac{7}{8}$ hours? Ans. $34\frac{1}{7}\frac{1}{2}$.

51. What cost $361\frac{11}{40}$ acres of land at $\$25\frac{3}{8}$ per acre?
Ans. $\$9167\frac{113}{320}$.

52. If $\frac{3}{8}$ of $\frac{8}{9}$ of a dollar buy one bushel of corn, what will $\frac{7}{9}$ of $\frac{9}{11}$ of a bushel cost? Ans. $\frac{7}{44}$ of a dollar.

53. How many square rods of land in a garden, which is $97\frac{5}{16}$ rods long, and $49\frac{3}{7}$ rods wide?
Ans. $4810\frac{1}{56}$ rods.

54. If $\frac{3}{8}$ of $\frac{4}{7}$ of $\frac{9}{11}$ of an acre of land cost one dollar, how much may be bought with $\frac{2}{3}$ of $\$18$?
Ans. $1\frac{47}{77}$ acres.

NOTE. The following questions are to exercise the foregoing rules.

55. What are the contents of a field $70\frac{7}{25}$ rods in length and $18\frac{3}{4}$ rods in breadth? Ans. 8A. 3R. $30\frac{1}{4}$p.

56. What are the contents of 10 boxes which are $7\frac{3}{4}$ feet long, $1\frac{3}{4}$ wide, and $1\frac{1}{4}$ feet in height?
Ans. $169\frac{17}{32}$ cubic feet.

57. From $\frac{7}{11}$ of an acre of land there were sold 20 poles and 200 square feet. What quantity remained?
Ans. 2R. 1p. $22\frac{3}{4}$ft.

58. What cost $\frac{1}{13}$ of an acre at $\$1.75$ per square rod?
Ans. $\$236.92\frac{4}{13}$.

59. What cost $\frac{3}{19}$ of a ton at $\$15\frac{3}{4}$ per cwt.?
Ans. $\$49.73\frac{13}{19}$.

60. What is the continued product of the following numbers $14\frac{2}{5}$, $11\frac{3}{7}$, $5\frac{1}{8}$, and $10\frac{1}{4}$? Ans. 9184.

61. From $\frac{7}{12}$ of a cwt. of sugar there was sold $\frac{4}{5}$ of it; what is the value of the remainder at $\$0.12\frac{3}{4}$ per lb.?
Ans. $\$3.57$.

62. What cost $19\frac{3}{7}$ barrels of flour at $\$7\frac{3}{8}$ per barrel ?
Ans. $\$143\frac{3}{7}$.

63. What cost $13\frac{83}{112}$ quintals of fish at $\$3\frac{3}{4}$ per quintal?
Ans. $\$51\frac{237}{448}$.

64. I have two parcels of land, one containing $7\frac{7}{10}$ acres, and the other $9\frac{11}{12}$ acres. What is their value at $\$78\frac{3}{8}$ per acre ? Ans. $\$1380.70\frac{5}{8}$.

65. From a quarter of beef weighing $175\frac{3}{4}$ lbs. I gave John Snow $\frac{2}{5}$ of it ; $\frac{3}{5}$ of the remainder I sold to John Cloon. What is the value of the remainder at $8\frac{3}{4}$ cents per lb. ? Ans. $\$2.04\frac{13}{15}$.

66. Alexander Green bought of John Fortune a box of sugar containing 475 lbs. for $\$30.00$. He sold $\frac{1}{5}$ of it at 8 cents per lb., and $\frac{2}{3}$ of the remainder at 10 cents per lb. What is the value of what still remains at $12\frac{1}{2}$ cents per lb., and what does Green make on his bargain ?

Ans. { Value of what remains $\$13.19\frac{1}{2}$.
Green's bargain, $\$16.97\frac{1}{2}$.

67. What cost $\frac{14}{101}$ of an acre at $\$14\frac{3}{7}$ per acre ?
Ans. $\$2.00$.

68. D. Sanborn's garden is $23\frac{1}{4}$ rods long and $13\frac{1}{4}$ rods wide, and is surrounded by a good fence $7\frac{1}{2}$ feet high. Now if he shall make a walk around his garden within the fence $7\frac{5}{12}$ feet wide, how much will remain for cultivation ? Ans. 1A. 3R. 7p. $85\frac{1381}{1764}$ft.

69. On $\frac{2}{3}$ of my field, I plant corn ; on $\frac{2}{3}$ of the remainder I sow wheat ; potatoes are planted on $\frac{3}{4}$ of what still remains, and I have left two small pieces, one of which is 3 rods square, and the other contains 3 square rods. How large is my field ? Ans. 1A. 0R. 29p.

70. Multiply $\frac{7}{8}$ of $\frac{8}{11}$ of $\frac{11}{14}$ by $\frac{5}{17}$ of $\frac{17}{19}$ of $\frac{19}{25}$. Ans. $\frac{1}{10}$.

Section 24.

DIVISION OF VULGAR FRACTIONS.

I. To divide a fraction by a whole number.

1. How many times will $\frac{5}{7}$ contain 9 ?

OPERATION.
$\frac{5}{7} \times \frac{1}{9} = \frac{5}{63}$ Ans.

To understand this question, we will suppose $\frac{5}{7}$ of an apple

were to be divided equally among 9 persons. Now, if we divide $\frac{1}{7}$ of an apple into 9 equal parts, there would be 63 parts, and each person would receive $\frac{1}{63}$; but there being $\frac{5}{7}$, each man will receive 5 times $\frac{1}{63} = \frac{5}{63}$ Ans. Hence we see the propriety of the following

RULE.

Multiply the whole number by the denominator of the fraction, and write the product under the numerator.

2. Divide $\frac{7}{11}$ by 12. Ans. $\frac{7}{132}$.
3. Divide $\frac{11}{12}$ by 8. Ans. $\frac{11}{96}$.
4. Divide $\frac{7}{8}$ by 12. Ans. $\frac{7}{108}$.
5. John Jones owns $\frac{1}{7}$ of a share in a railroad valued at $117; this he bequeaths to his five children. What part of a share will each receive? Ans. $\frac{1}{7}$.
6. Divide $\frac{8}{23}$ by 15. Ans. $\frac{8}{115}$.
7. Divide $\frac{6}{17}$ by 28. Ans. $\frac{3}{238}$.
8. James Page's estate is valued at $10,000, and he has given $\frac{2}{7}$ of it to the Seamen's Society; $\frac{1}{3}$ of the remainder he gave to his good minister; and the remainder he divided equally among his 4 sons and 3 daughters. What sum will each of his children receive? Ans. 680\frac{40}{147}$.

II. To divide a whole number by a fraction.

9. How many times will 13 contain $\frac{7}{8}$? Ans. $30\frac{1}{8}$.

OPERATION.

$\frac{13}{1} \times \frac{7}{8} = \frac{91}{8} = 30\frac{1}{8}$ Ans.

It is evident, that 13 will contain $\frac{1}{7}$, as many times as there are sevenths in 13, which are $7 \times 13 = 91$ times. Again, if 13 contain 1 seventh 91 times, it will contain 3 sevenths as many times as 91 will contain $3 = 30\frac{1}{8}$ Ans. Hence the following

RULE.

Multiply the whole number by the denominator of the fraction, and divide the product by the numerator.

10. Divide 18 by $\frac{7}{8}$. Ans. $20\frac{4}{7}$.
11. Divide 27 by $\frac{11}{12}$. Ans. $29\frac{5}{11}$.

12. Divide 23 by $\frac{1}{4}$. Ans. 92.
13. Divide 5 by $\frac{1}{5}$. Ans. 25.
14. Divide 12 by $\frac{3}{4}$. Ans. 16.
15. Divide 16 by $\frac{4}{5}$. Ans. 32.
16. Divide 100 by $\frac{17}{19}$. Ans. $111\frac{13}{17}$.
17. I have 50 square yards of cloth, how many yards, $\frac{3}{5}$ of a yard wide, will be sufficient to line it ?
Ans. $83\frac{1}{3}$ yards.
18. A. Poor can walk $3\frac{7}{11}$ miles in 60 minutes ; Benjamin can walk $\frac{9}{11}$ as fast as Poor. How long will it take Benjamin to walk the same distance ?
Ans. $73\frac{1}{3}$ minutes.

III. To divide a mixed number by an integer.

19. Divide $17\frac{3}{8}$ by 6. Ans. $2\frac{43}{48}$.

OPERATION.
$6)\ 1\ 7\frac{3}{8}$
$2\ 4\frac{3}{48}$

We divide 17 by 6, and find it is contained 2 times, which we write under the 17, and we have 5 remaining, which we multiply by 8, the denominator of the fraction ; and to the product we add the numerator, 3, and the amount is 43, this we write over the product of 6, the divisor, multiplied by the denominator, 8, = 48. The *rationale* of the above question is the same as of those in Rule I. of this section. Hence the following

RULE.

Divide the integers as in whole numbers, and if any thing remains, multiply it by the denominator of the fraction, and to the product add the numerator of the fraction, and write it over the product of the divisor, multiplied by the denominator.

20. Divide $17\frac{3}{5}$ by 7. Ans. $2\frac{18}{35}$.
21. Divide $18\frac{3}{7}$ by 8. Ans. $2\frac{17}{56}$.
22. Divide $27\frac{11}{12}$ by 9. Ans. $3\frac{11}{108}$.
23. Divide $31\frac{1}{10}$ by 11. Ans. $2\frac{21}{110}$.
24. Divide $78\frac{4}{5}$ by 12. Ans. $6\frac{17}{30}$.
25. Divide $189\frac{11}{15}$ by 4. Ans. $47\frac{13}{30}$.
26. Divide $107\frac{1}{12}$ by 3. Ans. $35\frac{25}{36}$.

27. Divide $\$ 17\frac{3}{7}$ among 7 men. Ans. $\$ 2\frac{24}{49}$.

28. Divide $\$ 106\frac{7}{9}$ among 8 boys. Ans. $\$ 13\frac{35}{72}$.

29. What is the value of $\frac{25}{72}$ of a dollar?

Ans. $\$ 0.34\frac{18}{18}$.

30. Divide $\$ 107\frac{7}{11}$ among 4 boys and 3 girls, and give the girls twice as much as the boys.

Ans. boy's share $\$ 10\frac{42}{55}$. Girl's share $\$ 21\frac{29}{55}$.

31. If $\$ 14$ will purchase $\frac{17}{20}$ of a ton of copperas, what quantity will $\$ 1$ purchase? Ans. 1cwt. 0qr. 24lbs.

IV. To divide one fraction by another.

32. Divide $\frac{7}{8}$ by $\frac{4}{5}$. Ans. $1\frac{11}{32}$.

OPERATION.

$\frac{7}{8} \times \frac{5}{4} = \frac{35}{32} = 1\frac{3}{32}$ Ans.

To understand the *rationale* of this process, we find the two factors of $\frac{4}{5}$, which are $\frac{4}{1}$ and $\frac{1}{5}$; for $\frac{4}{1}$ multiplied by $\frac{1}{5}$ are $\frac{4}{5}$, as is evident from a preceding rule. We now divide $\frac{7}{8}$ by $\frac{4}{1}$, which, by case I. of this section, will be $\frac{7}{8} \times \frac{1}{4} = \frac{7}{32}$. Again, we wish to divide $\frac{7}{32}$ by $\frac{1}{5}$. It is evident, that $\frac{7}{32}$ will contain $\frac{1}{5}$ nine times as often, as it will a unit, and it contains a unit $\frac{7}{32}$ times, therefore it contains $\frac{1}{5}$ nine times $\frac{7}{32} = \frac{5}{1} \times \frac{7}{32} = \frac{35}{32} = 1\frac{3}{32}$ Ans. In performing this question, it will be perceived, that the numerator of the dividend has been multiplied by the denominator of the divisor, and the denominator of the dividend by the numerator of the divisor. Hence the following

RULE.

Invert the divisor and proceed as in multiplication. If, however, there be mixed numbers in the question, they must be reduced to improper fractions, and compound fractions must be reduced to simple fractions.

33. Divide $\frac{7}{8}$ by $\frac{4}{5}$.

OPERATION.

$\frac{7}{8} \times \frac{7}{4} = \frac{49}{36} = 1\frac{13}{36}$ Ans.

34. Divide $7\frac{3}{4}$ by $3\frac{3}{7}$.

OPERATION.

$7\frac{3}{4} = \frac{31}{4}$, $3\frac{3}{7} = \frac{24}{7}$, $\frac{31}{4} \times \frac{7}{24} = \frac{217}{96} = 2\frac{25}{96}$ Ans.

35. Divide $\frac{7}{8}$ by $\frac{1}{4}$. Ans. $3\frac{1}{2}$.

36. Divide $1\frac{3}{5}$ by $\frac{11}{12}$. Ans. $\frac{52}{55}$.
37. Divide $\frac{2}{3}$ by $\frac{3}{10}$. Ans. $2\frac{2}{9}$.
38. Divide $\frac{9}{10}$ by $\frac{1}{7}$. Ans. $6\frac{3}{10}$.
39. Divide $\frac{4}{5}$ by $\frac{7}{11}$. Ans. $4\frac{2}{5}$.
40. Divide $7\frac{3}{8}$ by $4\frac{1}{2}$. Ans. $1\frac{23}{36}$.
41. Divide $3\frac{1}{2}$ by $7\frac{1}{2}$. Ans. $\frac{7}{15}$.
42. Divide $11\frac{1}{4}$ by $5\frac{3}{4}$. Ans. $2\frac{11}{152}$.
43. Divide $4\frac{3}{7}$ by $1\frac{7}{8}$. Ans. $2\frac{24}{112}$.
44. Divide $116\frac{3}{7}$ by $14\frac{1}{7}$. Ans. $8\frac{23}{99}$.
45. Divide $81\frac{1}{7}$ by $9\frac{1}{5}$. Ans. $8\frac{132}{161}$.
46. Divide $\frac{2}{3}$ of $\frac{7}{8}$ by $\frac{1}{7}$ of $\frac{2}{3}$. Ans. $18\frac{3}{8}$.

Section 25.

EXERCISES IN VULGAR FRACTIONS.

1. What are the contents of a board 9 inches long and 7 inches wide? Ans. 63 square inches.
2. What are the contents of a board $11\frac{3}{4}$ inches long, and $4\frac{1}{2}$ inches wide? Ans. $49\frac{1}{16}$ square inches.
3. How many square rods in a garden, which is $18\frac{3}{4}$ rods in length and $9\frac{7}{10}$ rods wide? Ans. $178\frac{5}{10}$ rods.
4. What cost $19\frac{3}{4}$ acres of land, at $\$17\frac{3}{4}$ per acre?
Ans. $\$350\frac{9}{16}$.
5. What cost $14\frac{7}{20}$ tons of coal at $\$7\frac{3}{4}$ per ton?
Ans. $\$111\frac{17}{80}$.
6. What cost $13\frac{11}{20}$ tons of hay at $\$8\frac{7}{8}$ per ton?
Ans. $\$120\frac{41}{160}$.
7. What cost $1\frac{7}{8}$ bushels of corn at $\$1\frac{7}{8}$ per bushel?
Ans. $\$3\frac{33}{64}$.
8. What is the value of $\frac{9}{16}$ of a dollar? Ans. $\$0.56\frac{1}{4}$.
9. What is the value of $\frac{17}{80}$ of a dollar? Ans. $\$0.21\frac{1}{4}$.
10. What is the value of $\frac{41}{160}$ of a dollar? Ans. $\$0.25\frac{5}{8}$.
11. What is the value of $\frac{33}{64}$ of a dollar? Ans. $\$0.51\frac{9}{16}$.
12. Bought a cask of molasses, containing $87\frac{1}{2}$ gallons; $\frac{3}{8}$ of it having leaked out, the remainder was sold at $27\frac{1}{2}$ cents per gallon; what was the sum received?
Ans. $\$15.03\frac{23}{32}$.

13. Bought of L. Johnson $7\frac{3}{8}$yds. of broadcloth, at $\$3\frac{7}{8}$ per yard, and sold it at $\$4\frac{3}{8}$ per yard; what was gained?
Ans. $\$3.68\frac{3}{4}$.

14. Bought a piece of land, that was $47\frac{5}{11}$ rods in length, and $29\frac{7}{16}$ in breadth; and from this land, there was sold to Abijah Atwood 5 square rods, and to Hazen Webster a piece that was 5 rods square; how much remains unsold? Ans. $1366\frac{83}{88}$ square rods.

15. Bought a tract of land that was 97 rods long and $48\frac{11}{16}$ rods wide; and from this I sold to John Ayer, a houselot, $18\frac{5}{12}$ rods long, and $14\frac{3}{8}$ rods wide; and the remainder of my purchase was sold to John Morse, at $\$3.75$ per square rod; what sum shall I receive?
Ans. $\$16717.30\frac{15}{32}$.

16. What are the contents of a box 8 feet long, 5 feet wide, and 3 feet high? Ans. 120 solid feet.

17. What are the contents of 10 boxes, each of which is $7\frac{3}{4}$ feet long, $4\frac{5}{12}$ feet wide, and $3\frac{5}{8}$ feet high?
Ans. $1312\frac{17}{144}$ feet.

18. Polly Brown has $\$17.87\frac{1}{2}$; half of this sum was given to the missionary society, and $\frac{2}{5}$ of the remainder she gave to the Bible society; what sum has she left?
Ans. $\$3.57\frac{1}{2}$.

19. What number shall be taken from $12\frac{3}{4}$, and the remainder multiplied by $10\frac{4}{7}$ that the product shall be 50?
Ans. $8\frac{13}{108}$.

20. What number must be multiplied by $7\frac{3}{5}$, that the product may be 20? Ans. $2\frac{13}{19}$.

21. Bought of John Dow $9\frac{7}{8}$ yards of cloth at $\$4.62\frac{1}{2}$ per yard; what was the whole cost? Ans. $\$45.67\frac{5}{16}$.

22. Bought of John Appleton $47\frac{3}{4}$ gallons of molasses for $\$12.37\frac{1}{2}$; what cost one gallon? what cost $12\frac{7}{8}$ gallons? Ans. $\$3.33\frac{5}{16}\frac{13}{64}$.

23. When $\$15.87\frac{1}{2}$ are paid for $12\frac{3}{8}$ bushels of wheat, what cost one bushel? what cost 11 bushels?
Ans. $\$14.11\frac{1}{9}$.

24. When $\$19.18\frac{3}{4}$ are paid for $3\frac{3}{8}$ cords of wood, what cost one cord? what cost $\frac{3}{8}$ of a cord?
Answer to the last, $\$2.13\frac{7}{36}$.

25. What are the contents of a box $8\frac{5}{12}$ feet long, $3\frac{1}{12}$ feet wide, and $2\frac{1}{12}$ feet high? Ans. $68\frac{11}{172}\frac{71}{8}$ feet.

Section 26.

DECIMAL FRACTIONS.

A Decimal Fraction is that, whose integer is always divided into 10, 100, 1000, &c. equal parts. Its denominator is always an unit, with as many ciphers annexed, as there are places in the given decimal. There is, therefore, no need of having the denominator expressed; for the value of the fraction is always known by placing a point before it, at the left hand, called the separatrix. Thus, .5 is $\frac{5}{10}$, .37 is $\frac{37}{100}$, .348 is $\frac{348}{1000}$.

Ciphers annexed to the right hand of decimals do not increase their value; for .4 or .40 or .400 are decimals having the same value, each being equal to $\frac{4}{10}$ or $\frac{2}{5}$; but when ciphers are placed on the left hand of a decimal, they decrease the value in a tenfold proportion. Thus .4 is $\frac{4}{10}$, or four tenths; but .04 is $\frac{4}{100}$, or four hundredths; and .004 is $\frac{4}{1000}$, or four thousandths. The figure next the separatrix is reckoned so many tenths; the next at the right, so many hundredths; the third is so many thousandths; and so on, as may be seen by the following

TABLE.

7	6	5	4	3	2	1.	2	3	4	5	6	7
Millions.	Hundreds of Thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.	Tenths.	Hundredths.	Thousandths.	Ten Thousandths.	Hundred Thousandths.	Millionths.

From this table it is evident, that in decimals, as well as in whole numbers, each figure takes its value by its distance from the place of units.

NOTE. If there be one figure in the decimal, it is so many tenths; if there be two figures, they express so many hundredths; if there be three figures, they are so many thousandths, &c.

NUMERATION OF DECIMAL FRACTIONS.

Let the pupil write the following numbers.

1. Three hundred seven, twenty-five hundredths.
2. Forty-seven, and seven tenths.
3. Eighteen and five hundredths.
4. Twenty-nine and three thousandths.
5. Forty-nine ten thousandths.
6. Eight and eight millionths.
7. Seventy-five and nine tenths.
8. Two thousand and two thousandths.
9. Eighteen and eighteen thousandths.
10. Five hundred five, and one thousand six millionths.

Section 27.

ADDITION OF DECIMALS.

1. Add together 5.018; 171,16; 88.133; 1113.6; .00456, and 14.178.

OPERATION.

5.018	= Five and eighteen thousandths.
171.16	= One hundred seventy-one, sixteen hundredths.
88.133	= Eighty-eight, and one hundred thirty-three thousandths.
1113.6	= One thousand one hundred thirteen, and six tenths.
.00456	= Four hundred fifty-six hundred thousandths.
14.178	= Fourteen, and one hundred seventy-eight thousandths.
1392.09356	= One thousand three hundred ninety-two, and nine thousand three hundred fifty-six hundred thousandths.

RULE.

Write the numbers under each other according to their value, add as in whole numbers, and point off from the right hand as many places for decimals, as there are in that number, which contains the greatest number of decimals.

2. Add together 171.61111 ; 16.7101 ; .00007 ; 71.0006, and 1.167895. Ans. 260.489775.

3. Add together .16711 ; 1.766 ; 76111.1 ; 167.1 ; .000007, and 1476.1. Ans. 77756.233117.

4. Add together 151.01 ; 611111.01 ; 16.5 ; 6.7 ; 46.1, and .67896. Ans. 611331.99896.

5. Add fifty-six thousand and fourteen thousandths, nineteen and nineteen hundredths, fifty-seven and forty-eight ten thousandths, twenty-three thousand and five and four tenths, and fourteen millionths. Ans. 79081.608814.

6. What is the sum of forty-nine and one hundred and five ten thousandths, eighty-nine and one hundred seven thousandths, one hundred and twenty-seven millionths, forty-eight ten thousandths ? Ans. 138.122427.

7. What is the sum of three and eighteen ten thousandths, one thousand five and twenty-three thousand forty-three millionths, eighty-seven and one hundred seven thousandths, forty-nine ten thousandths, and forty-seven thousand and three hundred nine hundred thousandths ?

Ans. 48095.139833.

Section 28.

SUBTRACTION OF DECIMALS.

RULE.

Let the numbers be so written that the separatrix of the subtrahend be directly under that of the minuend, that is, units under units, and tens under tens, &c.; subtract as in whole numbers, and point off so many places for decimals, as there are in that number, which contains the greatest number of decimals.

OPERATION.

1.	2.	3.	4.
11.078	47.117	46.13	87.107
9.81	8.78195	7.8915	1.11986
1.268	38.33505	382385	85.98714

5. From 81.35 take 11.678956. Ans. 69.671044.
6. From 1. take .876543. Ans. .123457.
7. From 100. take 99.111176. Ans. .888824.
8. From 87.1 take 5.6789. Ans. 81.4211.
9. From 100. take .001. Ans. 99.999.
10. From seventy-three take seventy-three thousandths. Ans. 72.927.
11. From three hundred sixty-five take forty-seven ten thousandths. Ans. 364.9953.
12. From three hundred fifty-seven thousand take twenty-eight and four thousand nine ten millionths. Ans. 356971.9995991.
13. From .875 take .4. Ans. .475.
14. From .3125 take .125. Ans. .1875.
15. From .95 take .44. Ans. .51.
16. From 3.7 take 1.8. Ans. 1.9.
17. From 8.125 take 2.6875. Ans. 5.4375.
18. From 9.375 take 1.5. Ans. 7.875.
19. From .666 take .041. Ans. .625.

Section 29.

MULTIPLICATION OF DECIMALS.

1. Multiply 18.72 by 7.1. Ans. 132.912.

OPERATION BY DECIMALS.

18.72
7.1
1872
13104
132.912 Ans.

BY VULGAR FRACTIONS.

$18\frac{72}{100} = \frac{1872}{100}$

$7.1 = \frac{71}{10}$

$\frac{1872}{100} \times \frac{71}{10} = \frac{132912}{1000} = 132\frac{912}{1000}$ Ans.

2. Multiply 15.12 by .012. Ans. .18144.

OPERATION BY DECIMALS.

15.12
.012
3024
1512
.18144 Ans.

BY VULGAR FRACTIONS.

$15\frac{12}{100} = \frac{1512}{100}$

$.012 = \frac{12}{1000}$

$\frac{1512}{100} \times \frac{12}{1000} = \frac{18144}{100000}$ Ans

Hence, we deduce the following

RULE.

Multiply as in whole numbers, and point off as many figures for decimals in the product, as there are decimals in the multiplicand and multiplier; but, if there be not so many figures in the product, as in the multiplicand and multiplier, supply the defect by prefixing ciphers.

3. Multiply 18.07 by .007. Ans. .12649.
4. Multiply 18.46 by 1.007. Ans. 18.58922.
5. Multiply .00076 by .0015. Ans. .00000114.
6. Multiply 11.37 by 100. Ans. 1137.
7. Multiply 47.01 by .047. Ans. 2.20947.
8. Multiply .0701 by. 0067. Ans. .00046967.
9. Multiply 47. by .47. Ans. 22.09.
10. Multiply eighty-seven thousandths by fifteen millionths. Ans. .000001305.
11. Multiply one hundred seven thousand and fifteen ten thousandths by one hundred seven ten thousandths. Ans. 1144.90001605.
12. Multiply ninety-seven ten thousandths by four hundred and sixty-seven hundredths. Ans. 3.886499.
13. Multiply ninety-six thousandths by ninety-six hundred thousandths. Ans. .00009216.
14. Multiply one million by one millionth. Ans. 1.
15. Multiply one hundred by fourteen ten thousandths. Ans. .14.
16. Multiply one hundred and one thousandth by ten thousand one hundred one hundred thousandths. Ans. .01020201.
17. Multiply one thousand fifty and seven ten thousandths, by three hundred five hundred thousandths. Ans. 3.202502135.
18. Multiply two million by seven tenths. Ans. 1400000.
19. Multiply four hundred and four thousandths by thirty and three hundredths. Ans. 12012.12012.
20. What cost 46lbs tea at $ 1.125 per lb. ? $ 51.75.
21. What cost 17.125 tons of hay at $ 18.875 per ton ? Ans. $ 323.234375.
22. What cost 18lbs. sugar at $ .125 per lb. ? Ans. $ 2.25.

Section 30.

DIVISION OF DECIMALS.

1. Divide $45.625 by 12.5. **2.** Divide $45\frac{625}{1000}$ by $12\frac{5}{10}$.

OPERATION BY DECIMALS.

12.5) 45.625 (3.65
375
812
750
625
625

BY VULGAR FRACTIONS.

$45\frac{625}{1000} = \frac{45625}{1000}$.

$12\frac{5}{10} = \frac{125}{10}$.

$\frac{45625}{1000} \times \frac{10}{125} = \frac{456250}{125000} = 3\frac{13}{20}$ Ans.

Hence the following

RULE.

Divide as in whole numbers, and point off as many decimals in the quotient, as the number of decimals in the dividend exceed those of the divisor; but, if the number of those in the divisor exceed that of the dividend, reduce the dividend to the same denomination as the divisor by annexing ciphers. And, if the number of decimals in the quotient and divisor together are not equal to the number in the dividend, supply the defect by prefixing ciphers to the quotient.

3. Divide 183.375 by 489. Ans. .375.
4. Divide 67.8632 by 32.8. Ans. 2.069.
5. Divide 67.56785 by .035. Ans. 1930.51.
6. Divide .567891 by 8.2. Ans. .069255.
7. Divide .1728 by 12. Ans. .0144.
8. Divide 172.8 by 1.2. Ans.
9. Divide 1728. by .12. Ans.
10. Divide .1728 by .12. Ans.
11. Divide 1.728 by 12. Ans.
12. Divide 17.28 by 1.2. Ans.
13. Divide 1728 by .0012. Ans.
14. Divide .001728 by 12. Ans.
15. Divide one hundred forty-seven and eight hundred twenty-eight thousandths by nine and seven tenths. Ans. 15.24.

16. Divide six hundred seventy-eight thousand seven hundred sixty-seven millionths by three hundred twenty-eight thousandths. Ans. 2.069.

Section 31.

REDUCTION OF DECIMALS.

I. To reduce a vulgar fraction to a decimal.

1. Reduce $\frac{5}{8}$ to a decimal.

OPERATION.

$$8)\overline{5.000}$$
$$.625$$

That the decimal .625 is equal to $\frac{5}{8}$, may be shown by writing it in a vulgar fraction and reducing it thus, $\frac{625}{1000} = \frac{5}{8}$ Ans.

NOTE. It is also evident, that .625 is equal to $\frac{5}{8}$, because the numerators have equal ratios to their denominators.

Hence the following

RULE.

Divide the numerator by the denominator, annexing one or more ciphers to the numerator, and the quotient will be the decimal required.

NOTE. It is not usually necessary, that decimals should be carried to more than six places.

2. Reduce $\frac{3}{4}$ to a decimal. Ans. .75.
3. Reduce $\frac{7}{8}$ to a decimal. Ans. .875.
4. What decimal fraction is equal to $\frac{7}{16}$? Ans. .4375.
5. Reduce $\frac{4}{11}$ to a decimal. Ans. .363636 +.
6. Reduce $\frac{5}{12}$ to a decimal. Ans. .416666 +.

II. Reduce compound numbers to decimals.

7. Reduce 8s. 6d. 3qr. to the decimal of a £.

OPERATION.

4	3.00
12	6.75
20	8.5625
	.428125

The 3 farthings are $\frac{3}{4}$ of a penny, and these, reduced to decimals, are .75 of a penny, which we annex to the pence, and proceed in the same manner with the other terms.

Hence the following

RULE.

Write the given numbers perpendicularly under each other for dividends, proceeding orderly from the least to the greatest; opposite to each dividend on the LEFT *hand, place such a number for a divisor, as will bring it to the next superior name, and draw a line between them. Begin at the highest, and write the quotient of each division, as decimal parts, on the* RIGHT *of the dividend next below it, and so on, until they are all divided; and the last quotient will be the decimal required.*

8. Reduce 15s. 6d. to the fraction of a £. Ans. .775.

9. Reduce 5cwt. 2qr. 14lb. to the decimal of a ton. Ans. .28125.

10. Reduce 3qr. 21lb. to the decimal of a cwt. Ans. .9375.

11. Reduce 6fur. 8rd. to the decimal of a mile. Ans. .775.

12. Reduce 3R. 19p. 167ft. 72in. to the decimal of an acre. Ans. .872595+.

NOTE 1. If it be required to reduce pounds, shillings, pence, and farthings, of the old New England currency, to dollars, cents, and mills; the pounds, shillings, &c. may be reduced to the decimal of a £; and if this decimal be multiplied by 10 and the product divided by 3, the quotient will be dollars and cents. But if the above decimal be multiplied by 10, and the product be divided by 4, the quotient will be the reduction of the old currency of New York to dollars and cents.

NOTE 2. If it be required to bring English sterling money to dollars and cents, let the pounds, &c. be reduced to the decimal of a penny; then divide this decimal by $\frac{9}{40}$, and the quotient is dollars and cents.

13. Change 18£. 15s. 6d. of the old New England currency, to dollars and cents.

OPERATION.

18£. 15s. 6d. = 18.775£.; $18.775 \times \frac{10}{3} = \$62.58\frac{1}{3}$ Ans.

14. Change 15£. 15s. 9d. of the old currency of New York, to dollars and cents.

OPERATION.

15£. 15s. 9d. = 15.7875£.; $15.7875 \times \frac{10}{4} = \$39.46.8\frac{3}{4}$ Ans.

15. Change 176£. 19s. 9d. sterling to United States currency. Ans. $786.61 +.

OPERATION.

176£. 19s. 9d. = 176.9875£.; $176.9876 \times \frac{4}{9} = \$786.61 +$.

III. To find the value of any given decimal in the terms of the integer.

16. What is the value of .9875£. ? Ans. 19s. 9d.

OPERATION.

.9875	
20	
19.7500	This question is performed by the same principle we adopted in finding the value of a vulgar fraction in the known parts of the integer.
12	
9.0000	

Hence the following

RULE.

Multiply the given decimal by that number which it takes of the next denomination to make one of that greater, and cut off as many places for a REMAINDER, *on the* RIGHT *hand, as there are places in the given decimal. Multiply the* REMAINDER *by the next lower denomination, and cut off for a remainder as before, and so proceed, until the decimal is reduced to the denomination required; the several denominations standing at the* LEFT *hand are the answers required.*

1. What is the value of .628125 of a £ ?
Ans. 12s. $6\frac{3}{4}$d.

2. What is the value of .778125 of a ton ?
Ans. 15cwt. 2qr. 7lb.

3. What is the value of .75 of an ell English ?
Ans. 3qr. 3na.

4. What is the value of .965625 of a mile ?
Ans. 7fur. 29rd.
5. What is the value of .94375 of an acre ?
Ans. 3R. 31p.
6. What is the value of .815625 of a pound Troy ?
Ans. 9oz. 15dwt. 18gr.
7. What is the value of .5555 of a pound apothecary's weight ? Ans. 6℥. 5℈. 0ʒ. 19$\frac{17}{25}$gr.

Section 32.

EXERCISES IN DECIMALS.

1. What is the value of 15cwt. 3qr. 14lb. of coffee at $9.50 per cwt. ? Ans. $150.81+.
2. What cost 17T. 18cwt. 1qr. 7lb. of potash at $53.80 per ton ? Ans. $963.86+.
3. What cost 37A. 3R. 16p. of land at $75.16 per acre ? Ans. $2844.80+.
4. What cost 15yd. 3qr. 2na. of cloth at $3.75 per yard ? Ans. $59.53+.
5. What cost 15$\frac{3}{8}$ cords of wood at 4.62\frac{1}{2}$ per cord ?
Ans. $71.10+.
6. What cost the construction of 17m. 6fur. 36rd. of railroad at $3765.60 per mile ? Ans. $67263.03+.
7. What cost 27hhd. 21gal. of temperance wine at 15.37\frac{1}{2}$ per hogshead ? Ans. $420.24+.
8. What are the contents of a pile of wood, 18ft. 9in. long, 4ft. 6in. wide, and 7ft. 3in. high ?
Ans. 611ft. 1242in.
9. What are the contents of a board 12ft. 6in. long, and 2ft. 9in wide ? Ans. 34ft. 54in.
10. Bought a cask of vinegar containing 25gal. 3qt. 1pt. at 0.37\frac{1}{2}$ per gallon ; what was the amount ?
Ans. $9.70+.
11. Bought a farm containing 144A. 3R. 30p. at 97.62\frac{1}{2}$ per acre ; what was the cost of the farm ?
Ans. $14149.52+.

12. Sold Joseph Punson 3T. 18cwt. 21lb. of salt hay, at $9.37½ per ton. He having paid me $20.25, what remains due? Ans. $16.40+.

13. If $\frac{7}{8}$ of a cord of wood cost $5.50, what cost one cord? What cost $7\frac{3}{4}$ cords? Ans. $48.71+.

Section 33.

SIMPLE INTEREST.

Interest is the compensation, which the borrower of money makes to the lender.

Principal is the sum lent.

Amount is the interest added to the principal.

Per Cent., a contraction of per centum, is the rate established by law, or that which is agreed on by the parties, and is so much for a hundred dollars for one year.

GENERAL RULE.

Let the per cent. be considered as a decimal of a hundred dollars, and multiply the principal by it, and the product is the interest for one year; but, if it be required to find the interest for more than one year, multiply the product by the number of years.

Note. The decimal for 6 per cent. is .06; for 7 per cent. .07; for 8 per cent. .08; for 9¼ per cent. .0925; for 2½ per cent. .025, &c. The decimals must be pointed off as in Multiplication of Decimal Fractions.

This rule is obvious from the fact, that the rate per cent. is such a part of every hundred dollars. Thus, 6 per cent. is $\frac{6}{100}$ of the principal.

Note. When no particular per cent. is named, 6 per cent. is to be understood, as it is the legal interest in the New England States generally. In New York the legal interest is 7 per cent.

1. What is the interest of $ 346 for one year ?

Ans. $ 20.76.

OPERATION.

346
.06
$20.76

There being two places of decimals in the multiplier, we point off two in the product.

2. What is the interest of $ 67.87 for 5 years ?

Ans. $ 20.36.

OPERATION.

67.87
.06
4.0722
5
$20.3610

There being two places of decimals in the multiplicand, and two in the multiplier, we point off four places in the product.

3. What is the interest of $ 197 for 1 year ?

Ans. $ 11.82.

4. What is the interest of $ 1728 for 3 years ?

Ans. $ 311.04.

5. What is the interest of $ 69 for 2 years ?

Ans. $ 8.28.

6. What is the interest of $ 1775 for 7 years ?

Ans. $ 745.50.

7. What is the interest of $ 987 for 10 years ?

Ans. $ 592.20.

8. Required the interest of $ 69.17 for 4 years.

Ans. $ 16.60.

9. Required the interest of $ 96.87 for 11 years.

Ans. $ 63.93.

10. Required the interest of $ 1.95 for 18 years.

Ans. $ 2.10.

11. Required the interest of $ 1789 for 20 years.

Ans. $ 2146.80.

12. Required the interest of $ 666.66 for 30 years.

Ans. $ 1199.98.

13. What is the amount of $ 98.50 for 5 years ?

Ans. $ 128.05.

14. What is the amount of $ 168.13 for 11 years ?

Ans. $ 279.09.

15. What is the amount of $ 75.75 for 17 years ?

Ans. $ 153.01.

16. Required the amount of $ 675.50 for 100 years.

Ans. $ 4728.50.

II. To find the interest for months, at six per cent.

RULE.

Multiply the principal by half the number of months, expressed decimally as a per cent.; that is, for 12 months multiply by .06; for 8 months multiply by .04; for 7 months .035; for 1 month .005, and point for decimals as in the last rule.

NOTE. It is obvious, that if half the number of months were 12, it would be 1 per cent. a month, that is, half the months will be the per cent. Q. e. d.

1. What is the interest of $ 486 for 10 months?

OPERATION.

4 8 6 principal.
.0 5 months decimal of the per cent.

$ 2 4.3 0 Ans.

2. What is the interest of $ 1728 for 18 months? Ans. $ 155.52.
3. What is the interest of $ 16.87 for 20 months? Ans. $ 1.68.
4. Required the interest of $ 118.15 for 30 months. Ans. $ 17.72.
5. Required the interest of $ 97.16 for 17 months. Ans. $ 8.25.
6. Required the interest of $ 789.87 for 23 months. Ans. $ 90.83.
7. Required the amount of $ 978.18 for 27 months. Ans. $ 1110.23.
8. Required the amount of $ 87.96 for 1 month. Ans. $ 88.39.
9. Required the amount of $ 81.81 for 100 months. Ans. $ 122.71.
10. Required the amount of $ 0.87 for 87 months. Ans. $ 1.24.

III. To find the interest for any sum for months and days, at 6 per cent.

RULE.

To one half of the months expressed decimally as in the last rule, annex one sixth of the days. With this multiply

the principal, and point off in the product as many decimals as there are in both factors; the first two figures at the right of the separatrix are cents, and the third is mills.

NOTE. If any other per cent. is required, proceed as before, and then divide the product by 6, and multiply the quotient by the rate required. The same result will be obtained if we multiply by the required rate, and divide the product by 6.

1. What is the interest of $57.50 for 10 months and 24 days? Ans. $3.105.

OPERATION.

57.50
.054
23000
28750
3.10500

We multiply by .054, because .05 is the rate per cent. for 10 months; and we annex the 4, because 4 is $\frac{1}{6}$ of the 24 days.

2. What is the interest of $178.75 for 17 months 17 days at 7 per cent.? Ans. $18.31.

OPERATION.

178.75
.087$\frac{1}{2}$
125125
143000
14895
6)1570020
261670
7
18.31690

3. What is the interest of $761.75 for 14 months and 18 days? Ans. $55.60.

4. What is the interest of $1728.19 for 17 months and 10 days? Ans. $149.77.

5. What is the interest of $88.96 for 16 months 6 days? Ans. $7.20.

6. What is the interest of $107.50 for 1 month 29 days? Ans. $1.05.

7. What is the interest of $87.25 for 20 months 5 days? Ans. $8.79.

8. What is the interest of $73.16 for 19 months 23 days? Ans. $7.23.

9. What is the interest of $1.71 for 24 months 2 days?
Ans. $0.20.

10. Required the interest of $100 for 100 months 1 day.
Ans. $50.01.

11. Required the interest of three dollars and five cents for 2 months and 2 days. Ans. $0.03.

IV. When the interest is required on any sum, from a certain day of the month in a year, to a particular day of a month in the same, or in another year, we adopt the following

RULE.

Find the time by placing the latest date in the upper line, and the earliest date under it. Let the year be placed first; and the number of months that have elapsed since the year commenced annexed at the right hand, and the day of the month next; then subtract the earlier from the later date, and the remainder is the time, for which the interest is required. Then proceed as in the last rule.

Note. Some Arithmeticians prefer reckoning the months by their *ordinal* number, as in operation 2d.

1. What is the interest of $172.50, from Sept. 25, 1840, to July 9, 1842? Ans. $18.51.5.

OPERATION 1st.

Y.	mo.	da.
1842	6	9
1840	8	25
1	9	14

$172.50
1.07½
120750
17250
5750
$18.51500

OPERATION 2d.

Y.	mo.	da.
1842	7	9
1840	9	25
1	9	14

It will be perceived, that the result in finding the time is the same in operation 2d, as in operation 1st.

2. What is the interest of $169.75, from Dec. 10, 1838, to May 5, 1841? Ans. $24.47.

3. What is the interest of $17.18, from July 29, 1837, to Sept. 1, 1841? Ans. $4.21.

4. What is the interest of $67.07, from April 7, 1839, to Dec. 11, 1841? Ans. $10.77.
5. Required the interest of $117.75, from Jan. 7, 1839, to Dec. 19, 1841. Ans. $20.84.
6. Required the interest of $847.15, from Oct. 9, 1839, to Jan. 11, 1843. Ans. $165.47.
7. Required the interest of $7.18, from March 1, 1841, to Feb. 11, 1842. Ans. $0.40.
8. What is the interest of $976.18, from May 29, 1842, to Nov. 25, 1845? Ans. $204.34.
9. I have John Smith's note for $144, dated July 25, 1839; what is due March 9, 1842? Ans. $166.65.
10. L. Johnson has J. Kimball's note, dated June 4, 1841, for $123; what is due to Johnson Dec. 7, 1843? Ans. $141.51.
11. George Cogswell has two notes against J. Doe; the first is for $375.83, and is dated Jan. 19, 1840; the other is for $76.19, dated April 23, 1841; what is the amount of both notes Jan. 1, 1842? Ans. $499.14.
12. What is the interest of $68.19, at 7 per cent., from June 5, 1840, to June 11, 1841? Ans. $4.85.
13. Required the amount of $79.15, from Feb. 17, 1839, to Dec. 30, 1842, at 7½ per cent. Ans. $102.11.
14. What is the amount of $89.96, from June 19, 1840, to Dec. 9, 1841, at 8½ per cent. Ans. $100.88.
15. A. Atwood has J. Smith's note for $325, dated June 5, 1839; what is due at 7¼ per cent., July 4, 1841? Ans. $374.02.
16. J. Ayer has D. How's note for $1728, dated Dec. 29, 1839; what is the amount Oct. 9, 1842, at 9 per cent.? Ans. $2160.00.
17. What is the interest of $976.18, from Jan. 29, 1841, to July 4, 1842, at 12 per cent.? Ans. $167.57.
18. What is the interest of $176.17, from June 19, 1839, to Sept. 7, 1843, at 9¾ per cent.? Ans. $72.42.
19. What is the amount of J. Turner's note for $87.25, dated June 1, 1841, to Dec. 17, 1843, at 5 per cent.? Ans. $98.35.
20. What is the amount of $379.78, from Dec. 3, 1808, to August 23, 1847, at 7¾ per cent.? Ans. $1519.48.

Section 34.

PARTIAL PAYMENTS.

I. When notes are paid within one year from the time they become due, it has been the usual custom to find the amount of the principal from the time it became due until the time of payment, and to find the amount of each indorsement from the time it was paid until settlement, and to subtract their sum from the amount of the principal.

1. $1234. Boston, Jan. 1, 1843.

For value received, I promise to pay John Smith, or order, on demand, one thousand two hundred thirty-four dollars, with interest. John Y. Jones.

Attest, Samuel Emerson.

On this note are the following indorsements.

March 1, 1843. Received ninety-eight dollars.
June 7, 1843. Received five hundred dollars.
Sept. 25, 1843. Received two hundred ninety dollars.
Dec. 8, 1843. Received one hundred dollars.

What remains due at the time of payment, Jan. 1, 1844? Ans. $293.12.

Principal		$ 1234.00
Interest for one year.		74.04
	Amount	1308.04
First payment	$98.00	
Interest for 10 months	4.90	
Second payment	500.00	
Interest for 6 months 24 days	17.00	
Third payment	290.00	
Interest for 3 months 6 days	4.64	
Fourth payment	100.00	
Interest for 23 days	38	
		$ 1014.92
Balance, remains due, Jan. 1, 1844		$293.12

2. $876.50. Boston, Sept. 25, 1842.

For value received, I promise to pay James Savage, or order, on demand, eight hundred seventy-six dollars fifty cents, with interest. Savage James.

Attest, John True.

On this note are the following indorsements.

Dec. 6, 1842. Received ninety-seven dollars.
Jan. 1, 1843. Received two hundred sixty-five dollars.
March 11, 1843. Received one hundred seventy dollars
July 4, 1843. Received seventy-nine dollars.

What remains due Aug. 6, 1843? Ans. $293.04.

3. $987.75. Danvers, Jan. 11, 1842.

For value received, we jointly and severally promise to pay Fitch Pool, or order, on demand two months from date, nine hundred eighty-seven dollars seventy-five cents, with interest after two months.

John T. Johnson.

Attest, Isaiah Webster. Samuel Jones.

On this note are the following indorsements.

May 1, 1842. Received three hundred dollars.
June 5, 1842. Received four hundred dollars.
Sept. 25, 1842. Received one hundred and fifty dollars.

What is due Dec. 13, 1842? Ans. $156.94.

4. $800. Bradford, July 4, 1842.

For value received, I promise to pay Leonard Johnson, or order, on demand, eight hundred dollars, with interest. Samuel Neverpay.

Attest, Enoch True.

On this note are the following indorsements.

Aug. 10, 1842. Received one hundred forty-four dollars.
Nov. 1, 1842. Received ninety dollars.
Jan. 1, 1843. Received four hundred dollars.
March 4, 1843. Received one hundred dollars.

What remains due June 1, 1843? Ans. $88.02.

II. In the United States' Court, and in most of the Courts of the several States, the following rule is adopted for estimating the interest on notes and bonds, when partial payments have been made.

RULE.

Compute the interest on the principal sum, from the time when the interest commenced to the time when the first payment was made, which exceeds, either alone or in conjunction with the preceding payments, if any, the interest at that time due; add that interest to the principal, and from the sum subtract the payment made at that time, together with the preceding payments, if any, and the remainder forms a new principal; on which compute and subtract the interest, as upon the first principal, and proceed in the same manner to the time of judgment.

This rule is illustrated in the following question.

1. $365.50. Lynn, Jan. 1, 1842.

For value received, I promise to pay John Dow, or order, on demand, three hundred sixty-five dollars fifty cents, with interest. John Smith.

Attest, Samuel Webster.

On this note are the following indorsements.

June 10, 1842. Received fifty dollars.
Dec. 8, 1842. Received thirty dollars.
Sept. 25, 1843. Received sixty dollars.
July 4, 1844. Received ninety dollars.
Aug. 1, 1845. Received ten dollars.
Dec. 2, 1845. Received one hundred dollars.

What remains due Jan. 7, 1847? Ans. $92.53.

OPERATION.

Principal carrying interest from Jan. 1, 1842, to June 10, 1842		$365.50
Interest from Jan. 1, 1842, to June 10, 1842, 5 months 9 days		9.68
	Amount	375.18
First payment, June 10, 1842		50.00
Balance for new principal		325.18

Balance for new principal (brought over)	325.18
Interest from June 10, 1842, to Dec. 8, 1842, 5 months 28 days	9.64
Amount	334.82
Second payment, Dec. 8, 1842	30.00
Balance for new principal	304.82
Interest from Dec. 8, 1842, to Sept. 25, 1843, 9 months 17 days	14.58
Amount	319.40
Third payment, Sept. 25, 1843	60.00
Balance for new principal	259.40
Interest from Sept. 25, 1843, to July 4, 1844, 9 months 9 days	12.06
Amount	271.46
Fourth payment, July 4, 1844	90.00
Balance for new principal	181.46
Interest from July 4, 1844, to Dec. 2, 1845, 16 months 28 days	15.36
Amount	196.82
Fifth payment, Aug. 1, 1845, {a sum less than the interest, $10.00	
Sixth payment, Dec. 2, 1845, {a sum greater than the interest, 100.00	
	110.00
Balance for new principal	86.82
Interest from Dec. 2, 1845, to Jan. 7, 1847, 13 months 5 days	5.71
Remains due Jan. 7, 1847	$92.53

2. $1000. Bradford, Jan. 10, 1836.

For value received, I promise to pay James Jones, or order, on demand with interest after three months, one thousand dollars. John Snow.

Attest, L. True.

On this note are the following indorsements.

July 4, 1836. Received one hundred dollars.
Jan. 1, 1837. Received two hundred dollars.
Sept. 25, 1838. Received three hundred dollars.
March 9, 1839. Received one hundred dollars.
April 7, 1840. Received two hundred and fifty dollars.

What is due Jan. 10, 1842? Ans. $232.26.

3. $1666. Newburyport, June 5, 1838.

For value received, I promise to pay John Boardman, or order, on demand, one thousand six hundred sixty-six dollars with interest. John J. Fortune.

Attest, T. Webster.

On this note are the following indorsements.

July 4, 1839. Received one hundred dollars.
Jan. 1, 1840. Received ten dollars.
July 4, 1840. Received fifteen dollars.
Jan. 1, 1841. Received five hundred dollars.
Feb. 7, 1842. Received six hundred fifty-six dollars.

What is due Jan. 1, 1843? Ans. $767.08.

Section 25.

COMMISSION AND BROKERAGE.

COMMISSION and BROKERAGE are compensations made to factors, brokers, and other agents, for their services, either for buying or selling goods.

NOTE. A factor is an agent, employed by merchants *residing in other places*, to buy, and sell, and to transact business on their account. A broker is an agent employed by merchants to transact business.

RULE.

The questions are performed in the same manner as in interest.

1. What is the commission on the sale of $5678 value of cotton goods, at 3 per cent.? Ans. $170.34.
2. A broker sells goods to the amount of $7896, at 2 per cent., what is his commission? Ans. $157.92.
3. My agent in Lowell has purchased goods for me to the amount of $1728, what is his commission, at 1½ per cent.? Ans. $25.92.
4. My factor advises me, that he has purchased, on my

account, 97 bales of cloth, at $15.50 per bale; what is his commission, at 2½ per cent.? Ans. $37.58+.

5. My agent, at New Orleans, informs me, that he has disposed of 500 barrels of flour at $6.50 per barrel, 88 barrels of apples at $2.75 per barrel, and 56 cwt. of cheese at $10.60 per cwt.? what is his commission, at 3¾ per cent.? Ans. $153.21.

NOTE. To estimate the duties on imported goods is performed in the same manner as interest, except when the duties are so much per ton, yard, &c.

6. What is the duty on $8000 value of imported goods, at 20 per cent.? Ans. $1600.

7. What is the duty on 50 tons of iron, at $30 per ton? Ans. $1500.

Section 36.

INSURANCE AND POLICIES.

INSURANCE is a security, by paying a certain sum to indemnify the secured against such losses, as shall be specified in the policy.

Policy is the name of the writ, or instrument, by which the contract or indemnity is effected between the parties.

RULE.

The same as in interest.

1. What is the premium on $868, at 12 per cent.? Ans. $104.16.

2. What is the premium on $1728, at 15 per cent.? Ans. $259.20.

3. A house, valued at $3500, is insured at 1¾ per cent.; what is the premium? Ans. $61.25.

4. A vessel and cargo, valued at $35000, is insured at 3¾ per cent.; now, if this vessel should be destroyed, what will be the actual loss to the insurance company? Ans. $33687.50.

Section 37.

STOCKS.

Stocks is the general name used for funds, established by government or individuals, in their corporate capacity, the value of which is often variable.

The method for computation is the same as in interest.

1. What must be given for 10 shares in the Boston and Portland Railroad, at 15 per cent. advance, shares being $100 each?

$100 × 10 = $1000; $1000 × 1.15 = $1150 Ans.

2. What must be given for 75 shares in the Lowell Railroad, at 25 per cent. advance, the original shares being $100 each? Ans. $9375.

3. What is the purchase of $8979 Bank stock at 12 per cent. advance? Ans. $10056.48.

4. What is the purchase of $1789 Bank stock at 9 per cent. below par? Ans. $1627.99.

Section 38.

BANKING.

When a note is discounted at a bank, the interest is taken at the time the note is given, and the interest is computed for 3 days more than the time specified in the note; that is, if the note is given for 60 days, the interest is taken for 63 days; for the law allows three days to the debtor, after the time has expired for payment, which are called days of *grace*. If, therefore, a note is given to the President and Directors of the Merrimack Bank for $100, to be paid in 60 days, the interest on the $100 is computed for 63, and taken from the *sum* of the note. So that the borrower receives only $98.95 for the note discounted.

1. What is the bank discount on $478, for 60 days?
Ans. $5.01+.
2. What is the bank discount on $780, for 30 days?
Ans. $4.29.
3. What is the bank discount on $1728, for 90 days?
Ans. $26.78+.
4. How much money should be received on a note of $1000, payable in 4 months, discounted at a bank, where the interest is 6 per cent.? Ans. $979.50.

Section 39.

DISCOUNT.

The object of discount is, to show what allowance should be made, when any sum of money is paid before it becomes due.

The present worth of any sum is the principal, that must be put at interest, to amount to that sum in the given time. That is, $100 is the *present worth* of $106, due one year hence; because $100 at 6 per cent. will amount to $106, and $6 is the discount.

Therefore when the interest is 6 per cent. the *present worth* is $\frac{100}{106}$ of the principal, and the *discount* is $\frac{6}{106}$ of the principal; and the same rule will hold good for any other per cent.

1. What is the present worth of $25.44, due one year hence? Ans. $24.00.

FIRST METHOD.

25.44
100
106)2544($24 Ans.
212
424
424

SECOND METHOD.

1.06)25.44($24 Ans.
212
424
424

From the above illustration, we deduce the following

RULE.

Divide the given sum by the amount of $ 1 *for the given rate and time, and the quotient will be the present worth. Or, multiply the given sum by* 100, *and divide the product by the amount of* $ 100 *for the given rate and time, and the quotient is the present worth.*

2. What is the present worth of $ 152.64, due one year hence? Ans. $ 144.00.
3. What is the present worth of $ 477.71, due four years hence? Ans. $ 385.25.
4. What is the present worth of $ 172.86, due 3 years 4 months hence? Ans. $ 144.05.
5. What is the present worth of $ 800, due 3 years 7 months and 18 days hence? Ans. $ 656.81+.
6. Samuel Heath has given his note for $ 375.75, dated Oct. 4, 1842, payable to John Smith, or order, Jan. 1, 1844; what is the real value of the note at the time given? Ans. $ 349.69+.
7. Bought a chaise and harness, of Isaac Morse, for $ 125.75, for which I gave him my note, dated Oct. 5, 1842, to be paid in six months; what is the present value of the note Jan. 1, 1843? Ans. $ 123.81+.
8. My tailor informs me, it will take 10 square yards of cloth to make me a full suit of clothes. The cloth I am about to purchase is $1\frac{3}{4}$ yards wide, and on spunging it will shrink 5 per cent. in width and length. How many yards of the above cloth must I purchase for my "*new suit*"? Ans. 6yd. 1qr. $1\frac{773}{2527}$na.

Section 40.

COMPOUND INTEREST.

The law specifies, that the borrower of money shall pay a certain number of dollars, called per cent., for the use of $ 100 for a year. Now, if this borrower does not pay to the lender this per cent. at the end of the year,

it is no more than just, that he should pay interest for the use of it, so long as he shall keep it in his possession; this is called Compound Interest.

1. What is the compound interest of $500 for 3 years? Ans. $95.50.

$500 = Principal.
1.06
30.00 = Interest for 1 year.
500.
530.00 = Amount for 1 year.
1.06
31.80 = Interest for second year.
530
561.80 = Amount for 2 years.
1.06
33.7080 = Interest for third year.
561.80
595.5080 = Amount for 3 years.
500
$95.50 = Compound interest for 3 years.

From the above process, we see the propriety of the following

RULE.

Find the interest of the given sum for one year, and add it to the principal; then find the interest of this amount for the next year; and so continue, until the time of settlement. Subtract the principal from the last amount, and the remainder is the compound interest.

2. What is the compound interest of $761.75 for 4 years? Ans. $199.94.

3. What is the amount of $67.25 for 3 years, at compound interest? Ans. $80.09+.

4. What is the amount of $78.69 for 5 years at 7 per cent.? Ans. $110.33.

5. What is the amount of $128 for 3 years 5 months and 18 days, at compound interest? Ans. $156 70.

6. What is the compound interest of $76.18 for 2 years 8 months 9 days? Ans. $12.96.

II. To find the amount of a note at compound interest, when there have been partial payments.

RULE.

Find the amount of the principal, and from it subtract the amount of the indorsements.

7. $144. Haverhill, Sept. 25, 1839.

For value received, I promise to pay Charles Northend, or order, on demand, one hundred forty-four dollars, with interest. John Small, Jr.

Attest, Q. Jones.

On this note are the following indorsements.

Jan. 1, 1840. Received thirty dollars.
June 30, 1841. Received eighty dollars.
Feb. 7, 1842. Received ten dollars.

What is due on the above note at compound interest, Oct. 4, 1842? Ans. $40.02.

OPERATION BY COMPOUND INTEREST.

Principal		$144.00
Interest from Sept. 25, 1839, to Oct. 4, 1842		27.76
	Amount	171.76
First payment	$30.00	
Interest from Jan. 1, 1840, to Oct. 4, 1842	5.23	
Second payment	80.00	
Interest from June 30, 1841, to Oct. 4, 1842	6.12	
Third payment	10.00	
Interest from Feb. 7, 1842, to Oct. 4, 1842	39	
	Amount	$131.74
Remains due, Oct. 4, 1842		$40.02

Section 41.

EQUATION OF PAYMENTS.

When several sums of money, to be paid at different times, are reduced to a mean time for the payment of the whole, without gain or loss to the debtor or creditor, it is called Equation of Payments.

1. John Jones owes Samuel Gray $100; $20 of which is to be paid in 2 months; $40 in 6 months; $30 in 8 months; and $10 in 12 months; what is the equated time for the payment of the whole sum?

Ans. 6mo. 12da.

OPERATION.

$20 × 2 = 40
$40 × 6 = 240
$30 × 8 = 240
$10 × 12 = 120
$100 100)640(6 mo.
600
40
30
100)1200(12 da.
1200

By analysis, $20 for 2 months is the same, as $40 for 1 month; and $40 for 6 months is the same, as $1 for 240 months; and $30 for 8 months is the same, as $1 for 240 months; and $10 for 12 months is the same, as $1 for 120 months; therefore, $1 for 40 + 240 + 240 + 120 = 640 months is the same, as $20 for 2 months, $40 for 6 months, $30 for 8 months, and $10 for 12 months; but $20 + $40 + $30 + $10 are $100; therefore, $1 for 640 months is the same, as $100 for $\frac{1}{100}$ of 640 months, which is 6 months and 12 days, as before. Hence the following.

RULE.

Multiply each payment by the time at which it is due, then divide the sum of the products by the sum of the payments, and the quotient will be the true time required.

2. John Smith owes a merchant, in Boston, $1000, $250 of which is to be paid in 4 months, $350 in 8

months, and the remainder in 12 months; what is the equated time for the payment of the whole sum?

Ans. 8mo. 18da.

Note. The following example will illustrate the method, the merchants practise to find the *medium* time of payment of goods sold on credit.

3. Purchased of James Brown, at sundry times, and on various terms of credit, as by the statement annexed. When is the *medium* time of payment?

Jan.	1,	a bill	amounting to	$360, on 3 months' credit.
Jan.	15,	do.	do.	186, on 4 months' credit.
March	1,	do.	do.	450, on 4 months' credit.
May	15,	do.	do.	300, on 3 months' credit.
June	20,	do.	do.	500, on 5 months' credit.

FORM OF STATEMENT.

Due April	1,	$360			
May	15,	$186	× 45 =	8370	
July	1,	$450	× 91 =	40950	
Aug.	15,	$300	× 136 =	40800	
Nov.	20,	$500	× 233 =	116500	
		1796		206620	(115 $\frac{20}{449}$ days.
				1796	
				2702	
				1796	
				9060	
				8980	
				80	

The medium time of payment will be 116 days from April 1, which will be July 25.

4. Sold S. Dana several parcels of goods, at sundry times, and on various terms of credit, as by the following statement.

Jan.	7, 1841,	a bill	amounting to	$375.60, on 4 months.
Apr.	18, 1841,	do.	do.	687.25, on 4 months.
June	7, 1841,	do.	do.	568.50, on 6 months.
Sept.	25, 1841,	do.	do.	300.00, on 6 months.
Nov.	5, 1841,	do.	do.	675.75, on 9 months.
Dec.	1, 1841,	do.	do.	100.00, on 3 months.

What is the equated time for payment of all the bills?

Ans. Dec. 24.

Section 42.

PROPORTION.

PROPORTION is the likeness or equalities of ratios. Thus, because 4 has the same ratio to 8, that 6 has to 12, we say such numbers are *proportionals*.

If, therefore, any four numbers whatever be taken, the first is said to have the same ratio or relation to the second, that the third has to the fourth, when the first number, or term, contains the second, as many times, as the third contains the fourth; or, when the second contains the first, as many times, as the fourth does the third. Thus, 9 has the same ratio to 3, that 12 has to 4, because 9 contains 3, as many times, as 12 does 4. And 10 has the same ratio to 5, that 12 has to 6, because 10 contains 5, as many times, as 12 does 6. Ratios are represented by colons; and equalities of ratios by double colons.

The first and third terms are called *antecedents*, and the second and fourth are called *consequents;* also, the first and fourth terms are called *extremes*, and the second and third are called *means*.

Whatever four numbers are proportionals, if their antecedents and consequents be multiplied or divided by the same numbers, they are still *proportionals;* and, if the terms of one proportion be multiplied or divided by the corresponding terms of another proportion, their products and quotients are still proportionals.

If the product of the extremes be equal to the product of the means, it is evident, that if any three of the four proportionals be given, the other may be obtained; for, if the product of the means be divided by one of the extremes, the quotient will be the other extreme; and, if the product of the extremes be divided by one of the means, the quotient will be the other mean. Hence the following

RULE.

State the question by making that number, which is of the same name or quality as the answer required, the third term;

then, if the answer required is to be greater than the third term, make the second term greater than the first; but if the answer is to be less than the third term, make the second less than the first.

Reduce the first and second terms to the lowest denomination mentioned in either, and the third term to the lowest denomination mentioned in it.

Multiply the second and third terms together, and divide their product by the first, and the quotient is the answer in the same denomination to which the third is reduced.

If any thing remains, after division, reduce it to the next lower denomination, and divide as before.

If either of the terms consists of fractions, state the question as in whole numbers, and reduce the mixed numbers to improper fractions, compound fractions to simple ones, and invert the first term, and then multiply the three terms continually together, and the product is the answer to the question. Or, the fractions may be reduced to a common denominator; and their numerators may be used as whole numbers. For when fractions are reduced to a common denominator, their value is as their numerators.

NOTE 1. It may be observed in Proportion, that the third term is the quantity, whose price or value is wanted, and that the second term is the value of the first; when, therefore, the second term is multiplied by the third, the product is as much more than the answer, as the first term is greater than unity; therefore, by dividing the product by the first term, we have the value of the quantity required.

NOTE 2. The pupil should perform every question by analysis, previous to his performing it by Proportion.

1. If 7lbs. of sugar cost 56 cents, what cost 36lbs. ?

lbs.	lbs.		cts.
7	36	::	56
	56		
	216		
	180		
7)	2016		
	$2.88 Ans.		

In stating this question, we make 56 cents the *third* term, because the answer will be in cents. And, as we perceive from the nature of the question, that the answer or fourth term will be *more* than 56 cents, we know, that of the other two terms, the *second* must be larger than the *first*, we therefore make 36lbs. the second term, and 7lbs. the first term.

To perform this question by *analysis*, we say, If 7lbs. cost 56 cents, one lb. will cost $\frac{1}{7}$ of 56 cents, which are 8 cents. Then, if 1lb. cost 8 cents, 36lbs. will cost 36 times as much; that is, 36 times 8 cents, which are $2.88 Ans. as before.

2. If 76 barrels of flour cost $456, what cost 12 barrels?

bar. bar. $
76 : 12 :: 456
456
72
60
48
$
76)5472(72 Ans.
532
152
152

As the answer to this question will be in dollars, we place $456 in the third term; and, as the answer or fourth term must be less than $456, because 12 barrels will cost less than 76 barrels, we must, of the other two terms, make the *less* the second term, and the *larger* the first term; that is, 12 barrels must be the second term, and 76 barrels the first term.

We analyze this question by saying, if 76 barrels cost $456, 1 barrel will cost $\frac{1}{76}$ of $456, which is $6. Then, if 1 barrel cost $6, 12 barrels will cost 12 times as much, that is, $72 Ans. as before.

3. If 3 men can dig a well in 20 days, how long would it take 12 men?

men. men. days.
12 : 3 :: 20
3
12)60(5 days, Ans.
60

As the answer will be in days, so the third term will be days. As 12 men will dig the well in less time than 3 men, therefore, the second term will be less than the first.

By analysis. If 3 men dig the well in 20 days, it will take one man 3 times as long, that is, 60 days. Again, we say, If one man dig the well in 60 days, 12 men would dig it in $\frac{1}{12}$ of 60 days, that is, 5 days, Ans. as before.

4. If 4lbs. of beef cost 36 cents, what cost 87lbs.?
Ans. $7.83.

5. What cost 9 gallons of molasses, if 63 gallons cost $14.49? Ans. $2.07

6. What cost 97 acres of land, if 19 acres can be obtained for $337.25? Ans. $1721.75.

7. If a man travel 319 miles in 11 days, how far will he travel in 47 days? Ans. 1363 miles.

8. If 7lbs. of beef will buy 4lbs. of pork, how much beef will be sufficient to buy 48lbs. of pork? Ans. 84lbs.

9. Paid for 87 tons of iron $5437.50, how many tons will $7687.50 buy? Ans. 123 tons.

10. When $120 are paid for 15 barrels of mackerel, what will be the cost of 79 barrels? Ans. $632.

11. If 9 horses eat a load of hay in 12 days, how many horses would it require to eat the hay in 3 days?
Ans. 36 horses.

12. When $5.88 are paid for 7 gallons of oil, what cost 27 gallons? Ans. $22.68.

13. When $10.80 are paid for 9lbs. of tea, what cost 147lbs.? Ans. $176.40.

14. What cost 27 tons of coal, when 9 tons can be purchased for $85.95? Ans. $257.85.

15. If 15 tons of lead cost $105, what cost 765 tons?
Ans. $5355.00.

16. If 16hhd. of molasses cost $320, what cost 176hhd?
Ans. $3520.00.

17. If 15cwt. 3qr. 17lb. of sugar cost $124.67, what cost 76cwt. 2qr. 19lb.? Ans. $601.09.

Note. When any of the terms is a compound number, it must be reduced to the lowest denomination mentioned in it; therefore, the hundred weights, quarters, &c., must be reduced to pounds, before the terms are multiplied and divided by each other.

18. If 7s. 6d. of the old Pennsylvania currency are equal to $1, what is the value of £76. 19s. 11d.?
Ans. $205.32⅔.

19. If 8s. of the old currency of New York are equal to $1, what is the value of £19. 19s. 8d.
Ans. $49.95+.

20. If 4s. 8d. of the old currency of South Carolina and Georgia are equal to $1, what is the value of £176. 18s. 4d.? Ans. $758.21+.

21. As 4s. 6d. sterling of the English currency are equal to one dollar in the United Sates, how many dollars are there in £769. 18s. 9d.? Ans. $3421.94+.

22. If the cars on the Boston and Portland Railroad go one mile in 2 minutes and 8 seconds, how long will they be in passing from Haverhill to Boston, the distance being 32 miles? Ans. 1h. 8min. 16sec.

23. If one acre of land cost $37.86, what cost 144A. 3R. 17p.? Ans. $5484.25+.

24. If a man travels 3m. 7fur. 18rd. in one hour, how far will he travel in 9h. 45min. 19sec.?
Ans. 38m. 2fur. 32+rd.

25. A fox is 96 rods before a greyhound, and, while the fox is running 15 rods the greyhound will run 21 rods; how far will the dog run before he can catch the fox?
Ans. 336 rods.

26. If 5 men can reap a field in 12 hours, how long would it take them if 4 men were added to their number? Ans. $6\frac{2}{3}$ hours.

27. Ten men engage to build a house in 63 days, but 3 of their number being taken sick, how long will it take the rest to complete the house? Ans. 90 days.

28. If a 4 cent loaf weighs 5 oz. when flour is $5 per barrel, what should it weigh when flour is $7.50 per barrel? Ans. $3\frac{1}{3}$ oz.

29. If 7 men can mow a field in ten days, when the days are 14 hours long, how long would it take the same men to mow the field, when the days are 13 hours long?
Ans. $10\frac{10}{13}$ days.

30. If 29lbs. of butter will purchase 40lbs. of cheese, how many pounds of butter will buy 79lbs. of cheese?
Ans. $57\frac{11}{40}$lb.

31. If $\frac{3}{4}$ of a yard cost $\frac{4}{5}$ of a dollar, what will $1\frac{1}{2}$ of a yard cost? Ans. $0.76\frac{7}{15}$.

STATEMENT. OPERATION.

$\frac{3}{4}$ yd. : $1\frac{1}{2}$ yd. :: $\frac{4}{5}$ $; $\frac{4}{3} \times \frac{11}{12} \times \frac{4}{5} = \frac{176}{180} = \$0.76\frac{7}{15}$ Ans.

NOTE. Let the pupil explain, why the first term is inverted in the operation.

32. If $\frac{7}{11}$ of a gallon of oil cost $\frac{9}{11}$ of a dollar, what cost $\frac{7}{8}$ of a gallon? Ans. $1.12\frac{1}{2}$.

STATEMENT. CANCELLED.

$\frac{7}{11}$ gal. : $\frac{7}{8}$ gal. :: $\frac{9}{11}$ $; $\frac{\cancel{11}}{\cancel{7}} \times \frac{\cancel{7}}{8} \times \frac{9}{\cancel{11}} = \frac{9}{8} = \$1.12\frac{1}{2}$ Ans.

33. If $4\frac{4}{5}$ yards of cloth cost $\$2\frac{7}{8}$, what will $19\frac{1}{2}$ yards cost? Ans. $\$11.50$.

STATEMENT. CANCELLED.

$$\text{yd. } 4\tfrac{4}{5} : \text{yd. } 19\tfrac{1}{2} :: \$2\tfrac{7}{8} ; \quad \frac{\$}{\not{39}} \times \frac{\not{39}}{2} \times \frac{23}{\not{8}} = \frac{23}{2} = \$11.50 \text{ Ans.}$$

34. If for $4\frac{7}{11}$ yards of velvet, there be received $11\frac{1}{2}$ yards of calico, how many yards of velvet will be sufficient to purchase 100 yards of calico?

Ans. $39\frac{8}{23}$ yards.

35. If $14\frac{1}{8}$ ells English of broadcloth will pay for $5\frac{8}{11}$ cwt. of sugar, how many yards will $25\frac{7}{11}$ cwt. buy?

Ans. 85yd. 3qr. $3\frac{32}{81}$na.

36. A certain piece of labor was to have been performed by 144 men in 36 days, but, a number of them having been sent away, the work was performed in 48 days; required the number of men discharged.

Ans. 36 men.

37 James can mow a certain field in 6 days, John can mow it in 8 days; how long will it take John and James both to mow it? Ans. $3\frac{3}{7}$ days.

38. Samuel can reap a field of barley in 9 hours; but, with the assistance of Alfred, he can reap it in 4 hours; how long would it take Alfred to reap it alone?

Ans. $7\frac{1}{5}$ hours.

39. A. Atwood can hoe a certain field in 10 days, but, with the assistance of his son Jerry, he can hoe it in 7 days; and he and his son Jacob can hoe it in 6 days; how long would it take Jerry and Jacob to hoe it together? Ans. $9\frac{3}{23}$ days.

40. Bought a horse for $\$75$; for what must I sell him to gain 10 per cent.?

$$\$100 : \$110 :: \$75 : \$82.50 \text{ Ans.}$$

41. Bought 40 yards of cloth at $\$5.00$ per yard; for what must I sell the whole amount to gain 15 per cent.?

Ans. $\$230.00$.

42. My chaise cost $\$175.00$, but, having been injured, I am willing to sell it on a loss of 30 per cent.; what should I receive? Ans. $\$122.50$.

43. Bought a cargo of flour on speculation at $\$5.00$ per barrel, and sold it at $\$6.00$ per barrel; what did I gain per cent.? Ans. 20 per cent.

44. Bought a hogshead of molasses for $ 15.00, but, it not proving so good as I expected, I sell it for $ 12.00; what do I lose per cent. ? Ans. 20 per cent.

45. Sold a pair of oxen for 20 per cent. less than their value, whereas, I might have sold them so as to have gained 20 per cent., and, by so doing, I have lost $ 60.00 ; what was the price for which they were sold ?

Ans. $ 120.00.

46. Bought a hogshead of molasses for $ 27.50, at 25 cents per gallon ; how much did it contain ?

Ans. 110 gallons.

47. A certain farm was sold for $ 1728, it being $ 15.75 per acre ; what was the quantity of land ?

Ans. 109A. 2R. 34$\frac{2}{7}$p.

Section 43.

COMPOUND PROPORTION.

COMPOUND PROPORTION is the method of performing by one operation, such questions as require two or more operations in Single Proportion.

1. If $ 100 will gain $ 6 in 12 months, what will $ 800 gain in 8 months ? Ans. $ 32.00.

$$\left.\begin{array}{l} \$\,100 \\ 12 \text{ months} \end{array}\right\} : \left\{\begin{array}{l} \$\,800 \\ 8 \text{ months} \end{array}\right. :: \$\,6 : \$\,32 \text{ Ans.}$$

OPERATION.

$$\frac{800 \times 8 \times 6}{100 \times 12} = \$\,32 \text{ Ans.}$$

The pupil will perceive, that the above operation is compounded of two statements in Single Proportion, which are as follows. If $ 100 gain $ 6 in one year, what will $ 800 gain in the same time ? Ans. $ 48.

OPERATION.

$$\$\,100 : \$\,800 :: \$\,6 : \$\,48.$$

Again, we say, If $800 will gain $48 in 12 months, what will the same sum gain in 8 months? Ans. $32.

OPERATION.

12 months : 8 months :: $48 : $32 Ans. as before.

This question may be analyzed in the following manner. We say, If $100 gain $6, $800 will gain 8 times as much, = $48. Again, we say, If 12 months gain $48, 1 month will gain $\frac{1}{12}$ of $48, = $4, and, if 1 month gain $4, 8 months will gain 8 times $4, = $32 Answer, as before.

NOTE. The pupil should analyze each question.

From the above illustrations, we deduce the following

RULE.

Make that number, which is of the same kind as the answer required, the third term; and, of the remaining numbers, take any two, that are of the same kind, and consider, whether an answer, depending upon these alone, would be greater or less than the third term, and place them as directed in Simple Proportion. Then take any other two, and consider, whether an answer, depending only upon them, would be greater or less than the third term, and arrange them accordingly; and so on until all are used. Multiply the continued product of the second terms by the third, and divide by the continued product of the first, and you produce the answer.

2. If $100 gain $6 in 12 months, in how many months will $800 gain $32. Ans. 8 months.
3. If $100 gain $6 in 12 months, how large a sum will it require to gain $32 in 8 months? Ans. $800.
4. If $800 gain $32 in 8 months, what is the per cent.? Ans. 6 per cent.
5. If 15 carpenters can build a bridge in 60 days, when the days are 15 hours long, how long will it take 20 men to build the bridge, when the days are 10 hours long? Ans. 67½ days.
6. If a regiment of soldiers, consisting of 939 men can eat 351 bushels of wheat in 3 weeks, how many soldiers will it require to eat 1404 bushels in 2 weeks? Ans. 5634 soldiers.

7. If 248 men, in $5\frac{1}{2}$ days of 11 hours each, dig a trench of 7 degrees of hardness, and $232\frac{1}{2}$ feet long, $3\frac{2}{3}$ feet wide, and $2\frac{1}{3}$ feet deep; in how many days of 9 hours each, will 24 men dig a trench of 4 degrees of hardness, and $337\frac{1}{2}$ feet long, $5\frac{1}{3}$ feet wide, and $3\frac{1}{2}$ feet deep?

Ans. 132 days.

Section 44.

COMPANY BUSINESS.

Company Business, or Fellowship, is a rule, by which merchants, and others in partnership, estimate their gain or loss in trade. It is of two kinds, *single* and *double*.

Single Fellowship is, when merchants in partnership employ their stock for *equal* times.

1. John Smith and Henry Grey enter into partnership for three years, with a capital of $6000, of which Smith puts in $4000, and Grey $2000. They gain $570. What is each man's share of the gain?

Ans. { Smith's gain $380.
{ Grey's gain $190.

Proof. $570.

As the whole stock is $6000, of which $4000 belongs to Smith, it is evident, that his share of the stock is $\frac{4000}{6000} = \frac{2}{3}$; and, as each man's gain is in proportion to his stock, $\frac{2}{3}$ of $570 = $380 is Smith's share of the gain. Grey's stock is $2000, therefore, $\frac{2000}{6000} = \frac{1}{3}$ of $570 = $190 is Grey's share of the gain.

Hence, to find any man's gain or loss in trade, we have the following

RULE.

Multiply the whole gain or loss by each man's FRACTIONAL PART *of the stock.*

2. Three merchants, A., B., and C., engage in trade. A. put in $6000, B. put in $9000, and C. put in

$5000. They gain $840. What is each man's share of the gain?

Ans. { A.'s gain $252. B.'s gain $378. C.'s gain $210.

Proof. $840.

3. A bankrupt owes Peter Parker $8750, James Dole $3610, and James Gage $7000. His effects sold at auction, amount to $6875; of this sum $375 are to be deducted for expenses, &c. What will each receive of the dividend?

Ans. { Parker 2937.75\frac{100}{121}$. Dole 1212.03\frac{62}{121}$. Gage 2350.20\frac{80}{121}$.

4. A merchant, failing in trade, owes A. $500, B. $386, C. $988, and D. $126. His effects are sold for $100. What will each man receive?

Ans. A. receives $25.00, B. $19.30, C. $49.40, D. $6.30.

Section 45.

DOUBLE FELLOWSHIP.

When merchants in partnership employ their stock for unequal times, it is called Double Fellowship.

1. Josiah Brown and George Dole trade in company Brown put in $600 for 8 months, and Dole put in $400 for 6 months. They gain $60. What is each man's share of the gain?

Operation by analysis. We say, $600 for 8 months is the same as $8 \times \$600 = \$4800$ for 1 month; and $400 for 6 months is the same as $6 \times \$400 = \$2400$ for 1 month. The question is, therefore, the same, as if Brown had put in $4800 and Dole $2400 for 1 month each. The whole stock would then be $\$4800 + \$2400 = \$7200$, and Brown's share of the gain would be $\frac{4800}{7200} = \frac{2}{3}$ of $\$60 = \$40$. Dole's share will be $\frac{2400}{7200} = \frac{1}{3}$ of $\$60 = \$20$. Hence the propriety of the following

RULE.

Multiply each man's stock by the time it continued in trade, and consider each product a numerator, to be written over their sum, as a common denominator; then multiply the whole gain or loss by each fraction, and the several products will be the gain or loss of each man.

2. A., B., and C. trade in company. A. put in $700 for 5 months; B. put in $800 for 6 months; and C put in $500 for 10 months. They gain $399. What is each man's share of the gain?

Ans. A.'s gain $105, B.'s gain $144, C.'s gain $150.

3. Leverett Johnson, William Hyde, and William Tyler, formed a connexion in business, under the firm of Johnson, Hyde, and Co.; Johnson at first put in $1000, and, at the end of 6 months, he put in $500 more. Hyde at first put in $800, and, at the end of 4 months, he put in $400 more, but, at the end of 10 months, he withdrew $500 from the firm. Tyler at first put in $1200, and, at the end of 7 months, he put in $300 more, and, at the end of 10 months, he put in $200. At the end of the year they found their net gain to be $1000. What is each man's share?

Ans. Johnson's gain 348.02\frac{338}{431}$, Hyde's 273.78\frac{82}{431}$, Tyler's 378.19\frac{11}{431}$.

4. George Morse hired of William Hale, of Haverhill, his best horse and chaise for a ride to Newburyport, for $3.00, with the privilege of one person's having a seat with him. Having rode 4 miles, he took in John Jones and carried him to Newburyport, and brought him back to the place from which he took him. What share of the expense should each pay, the distance from Haverhill to Newburyport being 15 miles?

Ans. Morse pays $1.90, Jones pays $1.10.

5. J. Jones and L. Cotton enter into partnership for one year. January 1, Jones put in $1000, but Cotton did not put in any until the first of April. What did he then put in to have an equal share with Jones at the end of the year? Ans. 1333.33\frac{1}{3}$.

Section 46.

DUODECIMALS.

Duodecimals are so called because they decrease by twelves, from the place of feet towards the right.

Inches are called *primes*, and are marked thus ′ ; the next division after is called *seconds*, marked thus ″ ; and so on.

1. Multiply 8 feet 6 inches by 3 feet 7 inches.

OPERATION.

8	6		
3	7		
25	6′		
4	11′	6″	
30	5′	6″	

As feet are the integers of units, it is evident, that feet multiplied by feet will produce feet; and, as inches are twelfths of a foot, the product of inches by feet will be twelfths of a foot. For the same reason, inches multiplied by inches will produce twelfths of an inch, or one hundred and forty-fourths of a foot. Hence we deduce the following

RULE.

Under the multiplicand write the same names or denominations of the multiplier; that is, feet under feet, inches under inches, &c. Multiply each term in the multiplicand, beginning at the lowest, by the feet of the multiplier, and write each result under its respective term, observing to carry a unit for every 12 *from each denomination to its next superior. In the same manner the multiplicand by the inches of the multiplier, and write the result of each term one place further towards the right of those in the multiplicand. Proceed in the same manner with the seconds, and all the rest of the denominations, and the sum of all the lines will be the product required.*

2. Multiply 8ft. 3in. by 7ft. 9in. Ans. 63ft. 11′ 3″.

3. Multiply 12ft. 9′ by 9ft. 11′. Ans. 126ft. 5′ 3″.

4. Multiply 14ft. 9′ 11″ by 6ft. 11′ 8″.

Ans. 103ft. 4′ 5″ 8‴ 4⁗.

5. Multiply 161ft. 8′ 6″ by 7ft. 10′. Ans. 1266ft. 8′ 7″.
6. Multiply 87ft. 1′ 11″ by 5ft. 7′ 5″.
Ans. 489ft. 8′ 0″ 2‴ 7⁗.
7. What are the contents of a board 18ft. long and 1ft. 10in. wide? Ans. 33ft.
8. What are the contents of a board 19ft. 8in. long and 2ft. 11in. wide? Ans. 57ft. 4′ 4″.
9. What are the contents of a floor 18ft. 9in. long and 10ft. 6in wide? Ans. 196ft. 10′ 6″.
10. How many square feet of surface are there in a room 14ft. 9in. long, 12ft. 6in. wide, and 7ft. 9in. high?
Ans. 791ft. 1′ 6″.
11. John Carpenter has agreed to make 12 shoe-boxes of boards that are one inch thick. The boxes are to be 3ft. 8in. long, 1ft. 9in. wide, and 1ft. 2in. high. How many square feet of boards will it require to make the boxes, and how many cubic feet will they contain?
Ans. 280 square feet; 66 cubic feet, 864 inches.
12. My garden is 18 rods long and 10 rods wide; a ditch is dug round it two feet wide and three feet deep, but the ditch not being of a sufficient breadth and depth, I have caused it to be dug one foot deeper and 1ft. 6in. wider. How many solid feet will it require to be removed? Ans. 7540 feet.

NOTE 1. A pile of wood, that is 8 feet long, 4 feet high, and 4 feet wide, contains 128 cubic feet, or a cord; and every cord contains 8 cord-feet; and, as 8 is $\frac{1}{16}$ of 128, every cord-foot contains 16 cubic feet; therefore, dividing the cubic feet in a pile of wood by 16, the quotient is the cord-feet; and, if cord-feet be divided by 8, the quotient is cords.

When wood is "corded" in a pile 4 feet wide, by multiplying its length by its height, and dividing the product by 4, the quotient is the cord-feet; and, if a load of wood be 8 feet long, and its height be multiplied by its width, and the product divided by 2, the quotient is the cord-feet.

NOTE 2. Small fractions are rejected in the operation.

13. How many cords of wood in a pile 56 feet long, 4 feet wide, and 5 feet 6 inches high? Ans. $9\frac{1}{8}$ cords.
14. How many cords of wood in a pile 23 feet 8 inches long, 4 feet wide, and 3 feet 9 inches high?
Ans. $2\frac{99}{128}$ cords.

15. How much wood in a pile 97 feet long, 3 feet 8 inches wide, and 7 feet high?

Ans. 19 cords $3\frac{2}{4}\frac{8}{5}$ feet.

16. If a pile of wood be 8 feet long, 3 feet 9 inches wide, how high must it be to contain one cord?

Ans. $4\frac{4}{1\,5}$ feet.

17. If a board be 1 foot 7 inches wide, how long must it be to contain 20 square feet?

Ans. 12 feet $7\frac{1}{1\,7}$ inches.

18. From a board 19 feet 7 inches long, I wish to slit off one square yard; how far from the edge must the line be drawn? Ans. $5\frac{1}{2}\frac{7}{5}$ inches.

19. I have a shed 19 feet 8 inches long, 14 feet 6 inches wide, and 7 feet 6 inches high; how many cords will it contain? Ans. 16 cords $5\frac{5}{8}$ feet $+$.

20. I have a room 12 feet long, 11 feet wide, and $7\frac{1}{2}$ feet high; in it are 2 doors, 6 feet 6 inches high, and 30 inches wide, and the mop-boards are 8 inches high; there are 3 windows, 3 feet 6 inches wide, and 5 feet 6 inches high; how many square yards of paper will it require to cover the walls?

Ans. $25\frac{29}{10\,8}$ square yards.

Section 47.

INVOLUTION.

Involution is the raising of powers from any given number, as a root.

A power is a quantity produced by multiplying any given number, called a root, a certain number of times continually by itself; thus,

$3 = 3$ is the first power of $3 = 3^1$.
$3 \times 3 = 9$ is the second power of $3 = 3^2$.
$3 \times 3 \times 3 = 27$ is the third power of $3 = 3^3$.
$3 \times 3 \times 3 \times 3 = 81$ is the fourth power of $3 = 3^4$.

The number denoting the power is called the *index*, or *exponent*, of the power. Thus, the fifth power of 2 is 32, or 2^5; the third power of 4 is 64, or 4^3.

To raise any number to any power required, we adopt the following

RULE.

Multiply the given number continually by itself, till the number of multiplications be one less, than the index of the power to be found, and the last product will be the power required.

1. What is the 3rd power of 5? $5 \times 5 \times 5 = 125$ Ans.
2. What is the 6th power of 4? Ans. 4096.
3. What is the 4th power of 3? Ans. 81.
4. What is the 1st power of 17? Ans. 17.
5. What is the 0 power of 63? Ans. 1.

Section 48.

EVOLUTION,

OR THE EXTRACTION OF ROOTS.

EVOLUTION, or the reverse of involution, is the extraction or finding the roots of any given power.

The *root* is a number, whose continued multiplication into itself produces the power, and is denominated the square, cube, biquadrate, or second, third, fourth, &c., power, equal to that power.

Thus, 4 is the square root of 16, because, $4 \times 4 = 16$; and 3 is the cube root of 27, because, $3 \times 3 \times 3 = 27$; and so on.

Roots, which approximate, are *surd roots*; and those, which are perfectly accurate, are called *rational roots*.

EXTRACTION OF THE SQUARE ROOT.

1. What is the square root of 625?

To illustrate this question, we will suppose, that we

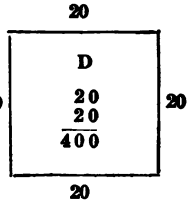
have 625 tile, each of which is one foot square; we wish to know the side of a square room, whose floor they will pave or cover. If we find a number multiplied into itself, that will produce 625, that number will be the side of a square room, which will require 625 tiles to cover its floor. We perceive that our number (625) consists of three figures, therefore, there will be two figures in the root; for the product of any two numbers can have, at most, but just so many figures, as there are in both factors, and, at least, but one less. We will, therefore, for convenience, divide our number (625) into two parts, called periods, writing a point over the right hand figure of each period; thus, 6̇25̇. We now find, that the greatest square number in the left hand period, 6 (hundred), is 4 (hundred); and that its root is 2, which we write in the quotient (see operation). As this 2 is in the place of tens, its value must be 20 and its square 400.

OPERATION.

```
 6̇25̇ (25 Ans.
 400
45)225
 225
```

Let this be represented by a square, whose sides measure 20 feet each, and whose contents will, therefore, be 400 square feet. (See figure 1.) We now subtract 400 from 625, and there remains 225 square feet, to be arranged on two sides of figure 1, in order that its form may remain square. We therefore double the root 20, one of the sides, and it gives the length of the two sides to be enlarged; viz. 40. We then inquire, how many times 40, as a divisor, is contained in the dividend, and find it to be 5 times; this we write in the root, and also in the divisor.

FIG. 1.

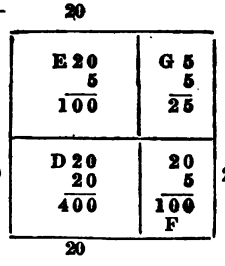


This 5 is the breadth of the addition to our square. (See figure 2.) And this breadth, multiplied by the length of the two additions (40) gives the contents of the two figures, E and F, 200 square feet, which is 100 feet for each.

There now remains the figure G, to complete the square, each side of which is 5 feet; it being equal to

the breadth of the additions E and F. Therefore, if we square 5, we have the contents of the last addition, G = 25. It is on account of this last addition, that the last figure of the root is placed in the divisor. If we now multiply the divisor, 45, by the last figure in the root (5), the product will be 225, which is equal to the remaining feet, after we have formed our first square, and equal to the additions E, F, and G, in figure 2. We therefore perceive, that figure 2 may represent a floor 25 feet square, containing 625 square feet. From the above, we infer the following

FIG. II.



D contains 400 square feet.
E do. 100 do. do.
F do. 100 do. do.
G do. 25 do. do.
Proof. 625
or,
$25 \times 25 = 625$.

RULE.

1. *Distinguish the given number into periods of two figures each, by putting a point over the place of units, another over the place of hundreds, and so on, which points show the number of figures the root will consist of.*

2. *Find the greatest square number in the first or left hand period, place the root of it at the right hand of the given number, (after the manner of a quotient in division,) for the first figure of the root, and the square number under the period, and subtract it therefrom, and to the remainder bring down the next period for a dividend.*

3. *Place the double of the root already found, on the left hand of the dividend for a divisor.*

4. *Seek how often the divisor is contained in the dividend, (except the right hand figure,) and place the answer in the root for the second figure of it, and likewise on the right hand of the divisor. Multiply the divisor with the figure last annexed by the figure last placed in the root, and subtract the product from the dividend. To the remainder join the next period for a new dividend.*

5. *Double the figures already found in the root for a new*

divisor, (or, bring down your last divisor for a new one, doubling the right hand figure of it,) and from these find the next figure in the root, as last directed, and continue the operation in the same manner, till you have brought down all the periods.

Note 1. If, when the given power is pointed off, as the power requires, the left hand period should be deficient, it must nevertheless stand as the first period.

Note 2 If there be decimals in the given number, it must be pointed both ways from the place of units. If, when there are integers, the first period in the decimals be deficient, it may be completed by annexing so many ciphers as the power requires. And the root must be made to consist of so many whole numbers and decimals, as there are periods belonging to each; and when the periods belonging to the given numbers are exhausted, the operation may be continued at pleasure by annexing ciphers.

Note 3. If it be required to extract the square root of a vulgar fraction, reduce the fraction to its lowest terms, then extract the square root of the numerator for a new numerator, and of the denominator for a new denominator; or, reduce the vulgar fraction to a decimal, and extract its root.

2. What is the square root of 148996?

OPERATION.

$$\begin{array}{r} 1\dot{4}\dot{8}\dot{9}\dot{9}6\dot{}\,(386 \\ 9 \\ \hline 68)\overline{5}89 \\ 544 \\ \hline 766)\overline{4596} \\ 4596 \\ \hline \end{array}$$

3. What is the square root of 23804641? Ans. 4879.
4. What is the square root of 10673289? Ans. 3267.
5. What is the square root of 20894041? Ans. 4571.
6. What is the square root of 1014049? Ans. 1007.
7. What is the square root of 516961? Ans. 719.
8. What is the square root of 182329? Ans. 427.
9. What is the square root of 61723020.96? Ans. 7856.4.
10. What is the square root of 9754.60423716? Ans. 98.7654.
11. What is the square root of $\frac{3721}{1369}$? Ans. $\frac{61}{37}$.

12. What is the square root of $\frac{1849}{12769}$? Ans. $\frac{43}{113}$.
13. What is the square root of $\frac{49}{529}$? Ans. $\frac{7}{23}$.
14. What is the square root of $\frac{196}{625}$? Ans. $\frac{14}{25}$.
15. What is the square root of $60\frac{1}{16}$? Ans. $7\frac{3}{4}$.
16. What is the square root of $28\frac{17}{64}$? Ans. $5\frac{3}{8}$.
17. What is the square root of $47\frac{17}{64}$? Ans. $6\frac{7}{8}$.

APPLICATION OF THE SQUARE ROOT.

18. A certain general has an army of 226576 men; how many must he place rank and file to form them into a square? Ans. 476.

NOTE. In a right angle triangle, the square of the longest side is equal to the sum of the squares of the other two sides.

19. What must be the length of a ladder to reach to the top of a house 40 feet in height; the bottom of the ladder being placed 9 feet from the sill? Ans. 41 feet.
20. Two vessels sail from the same port; one sails due north 360 miles, and the other due east 450 miles; what is their distance from each other?
Ans. 576.2+ miles.
21. If a pipe, 2 inches in diameter, will fill a cistern in $20\frac{1}{4}$ minutes, how long would it take a pipe, that is 3 inches in diameter? Ans. 9 minutes.
22. If an anchor, which weighs 2000 lbs., requires a cable 3 inches in diameter, what should be the diameter of a cable, when the anchor weighs 4000lbs.?
Ans. 4.24+ inches.
23. How large a square stick may be hewn from a round one, which is 30 inches in diameter?
Ans. 21.2+ inches square.
24. John Snow's dwelling is 60 rods north of the meetinghouse, James Briggs' is 80 rods east of the meetinghouse, Samuel Jenkins' is 70 rods south, and James Emerson's 90 rods west of the meetinghouse; how far will Snow have to travel to visit his three neighbours, and then return home? Ans. 428.4+ rods.

Section 49.

EXTRACTION OF THE CUBE ROOT.

A Cube is a solid, bounded by six equal squares.

A number is said to be cubed, when it is multiplied into its square.

To extract the cube root, is to find a number, which, being multiplied into its square, will produce the given number.

The extraction of this root has been illustrated by mathematicians in various ways. But it is believed, that Robert Record, Esquire, of London, in his Arithmetic published in 1673, was among the first, who illustrated this rule by the use of various diagrams and blocks. The same thing, with but *little* variation, has been done by several arithmeticians in our own country.

The Rule for extracting the root depends on the following

THEOREM.

If any line or number be divided into two parts, the cube of the whole line or number, is equal to the cube of the greater part, plus the square of the greater part multiplied by 3 times the less part, plus the square of the less part multiplied by 3 times the larger part, plus the cube of the less part.

To illustrate this Theorem, let 27 be divided into two parts, 20 and 7. Then, by the hypothesis, the cube of 27 is equal to the cube of 20, plus the square of 20 multiplied by 3 times 7, plus the square of 7 multiplied by 3 times 20, plus the cube of 7.

OPERATION.

Cube of 27	= 19683
Cube of 20	= 8000
Square of 20 multiplied by 3 times 7	= 8400
Square of 7 multiplied by 3 times 20	= 2940
Cube of 7	= 343
Proof.	= 19683.

Hence the following

RULE.

1. *Separate the given number into periods of three figures each, by putting a point over the unit figure, and every third figure beyond the place of units.*

2. *Find by the table the greatest cube in the left hand period, and put its root in the quotient.*

3. *Subtract the cube, thus found, from this period, and to the remainder bring down the next period; call this the dividend.*

4. *Multiply the square of the quotient by* 300, *calling it the triple square; multiply also the quotient by* 30, *calling it the triple quotient; the sum of these call the divisor.*

5. *Find how many times the divisor is contained in the dividend, and place the result in the quotient.*

6. *Multiply the triple square by the last quotient figure, and write the product under the dividend; multiply the square of the last quotient figure by the triple quotient, and place this product under the last; under all, set the cube of the last quotient figure, and call their sum the subtrahend.*

7. *Subtract the subtrahend from the dividend, and to the remainder bring down the next period for a new dividend, with which proceed as before, and so on, till the whole is completed.*

NOTE 1. The same rule must be observed for continuing the operation, and pointing for decimals, as in the square root.

NOTE 2. In inquiring how many times the dividend will contain the divisor, we must sometimes make an allowance of two or three units. See National Arithmetic, page 205.

1. What is the cube root of 78402752 ?

OPERATION.

$\dot{7}8\dot{4}0\dot{2}75\dot{2}(428$		$4\times4\times300=$	4800
64		$4\times30=$	120
4920)14402	=1st dividend.	1st divisor.=	4920
9600		$4800\times2=$	9600
480		$120\times2\times2=$	480
8		$2\times2\times2=$	8
10088	=1st subtrahend.	1st subtrahend.=	10088

530460)4314752=2d dividend. $42\times42\times300=529200$
4233600 $42\times30=1260$
80640 2d divisor.= 530460
512 $529200\times8=4233600$
4314752=2d subtrahend. $1260\times8\times8=80640$
$8\times8\times8=512$
2d subtrahend.=4314752

2. What is the cube root of 74088 ? Ans. 42.
3. What is the cube root of 185193 ? Ans. 57.
4. What is the cube root of 80621568 ? Ans. 432.
5. What is the cube root of 176558481 ? Ans. 561.
6. What is the cube root of 257259456 ? Ans. 636.
7. What is the cube root of 1860867 ? Ans. 123.
8. What is the cube root of 1879080904 ? Ans. 1234.
9. What is the cube root of 41673648.563 ? Ans. 346.7.
10. What is the cube root of 48392.1516051 ? Ans. 78.51.
11. What is the cube root of 8.144865728 ? Ans. 2.012.
12. What is the cube root of $\frac{729}{4096}$? Ans. $\frac{9}{16}$.
13. What is the cube root of $49\frac{8}{27}$? Ans. $3\frac{2}{3}$.
14. What is the cube root of $166\frac{2}{3}$? Ans. $5\frac{1}{2}$.
15. What is the cube root of $85\frac{33}{125}$? Ans. $4\frac{2}{5}$.

APPLICATION OF THE CUBE ROOT.

Spheres are to each other, as the cubes of their diameter.

Cones are to each other, as the cubes of their altitudes or bases.

All similar solids are to each other, as the cubes of their homologous sides.

16. If a ball, 4 inches in diameter, weighs 50lbs., what is the weight of a ball 6 inches in diameter ? Ans. 168.7+ lbs.
17. If a sugar loaf, which is 12 inches in height, weighs 16lbs., how many inches may be broken from the base, that the residue may weigh 8lbs. ? Ans. 2.5+ in.

18. If an ox, that weighs 800lbs., girts 6 feet, what is the weight of an ox that girts 7 feet? Ans. 1270.3lbs.

19. If a tree, that is one foot in diameter, make one cord, how many cords are there in a *similar* tree, whose diameter is two feet? Ans. 8 cords.

20. If a bell, 30 inches high, weighs 1000lbs., what is the weight of a bell 40 inches high? Ans. 2370.3lbs.

21. If an apple, 6 inches in circumference, weighs 16 ounces, what is the weight of an apple 12 inches in circumference? Ans. 128 ounces.

Section 50.

GEOMETRICAL PROBLEMS.

1. To find the area of a square or parallelogram.

RULE. *Multiply the length by the breadth, and the product is the superficial contents.*

2. To find the area of a rhombus or rhomboid.

RULE. *Multiply the length of the base by the perpendicular height.*

3. To find the area of a triangle.

RULE. *Multiply the base by half the perpendicular height; or, add the three sides together; then take half of that sum, and out of it subtract each side severally; multiply the half of the sum and these remainders together, and the square root of this product will be the area of the triangle.*

4. Having the diameter of a circle given, to find the circumference.

RULE. *Multiply the diameter by* 3.141592, *and the product is the circumference.*

NOTE. The exact proportion, which the diameter of a circle bears to the circumference, has never been discovered, although some mathematicians, have carried it to 200 places of decimals If the diameter of a circle be 1 inch, the circumference will be 3.141592653 589793238462643383279502884197169399375105820974944592307816406286 208998628034825342117067982148086513282306647093844609550582 2317253594081284802 inches nearly.

5. Having the diameter of a circle given, to find the side of an equal square.

RULE. *Multiply the diameter by* .886227, *and the product is the side of an equal square.*

6. Having the diameter of a circle given, to find the side of an equilateral triangle inscribed.

RULE. *Multiply the diameter by* .707016, *and the product is the side of a triangle inscribed.*

7. Having the diameter of a circle given, to find the area.

RULE. *Multiply the square of the diameter by* .785398, *and the product is the area. Or, multiply half the diameter by half the circumference, and the product is the area.*

8. Having the circumference of a circle given, to find the diameter.

RULE. *Multiply the circumference by* .31831, *and the product is the diameter.*

9. Having the circumference of a circle given, to find the side of an equal square.

RULE. *Multiply the circumference by* .282094, *and the product is the side of an equal square.*

10. Having the circumference of a circle given, to find the side of an equilateral triangle inscribed.

RULE. *Multiply the circumference by* .2756646, *and the product is the side of an equilateral triangle inscribed.*

11. Having the circumference of a circle given, to find the side of an inscribed square.

RULE. *Multiply the circumference by* .225079, *and the product is the side of a square inscribed.*

12. To find the contents of a cube or parallelopipedon.

RULE. *Multiply the length, height, and breadth, continually together, and the product is the contents.*

13. To find the solidity of a prism.

RULE. *Multiply the area of the base, or end, by the height.*

14. To find the solidity of a cone or pyramid.

RULE. *Multiply the area of the base by $\frac{1}{3}$ of its height.*

15. To find the surface of a cone.

RULE. *Multiply the circumference of the base by half its slant height.*

16. To find the solidity of the frustum of a cone, or pyramid.

RULE. *Multiply the diameters of the two bases together, and to the product add $\frac{1}{3}$ of the square of the difference of the diameters; then multiply this sum by .785398, and the product will be the mean area between the two bases; lastly, multiply the mean area by the length of the frustum, and the product will be the solid contents.*

Or, find when it would terminate in a cone, and then find the contents of the part supposed to be added, and take it away from the whole.

17. To find the solidity of a sphere or globe.

RULE. *Multiply the cube of the diameter by .5236.*

18. To find the convex surface of a sphere or globe.

RULE. *Multiply its diameter by its circumference.*

19. To find the contents of a spherical segment.

RULE. *From three times the diameter of the sphere, take double the height of the segment; then multiply the remainder by the square of the height, and the product by the decimal .5236 for the contents; or to three times the square of the radius of the segment's base, add the square of its*

height; then multiply the sum by the height, and the product by .5236 for the contents.

20. To find how large a cube may be cut from any given sphere, or be inscribed in it.

RULE. *Square the diameter of the sphere, divide that product by* 3, *and extract the square root of the quotient for the answer.*

21. To find the number of gallons, &c., in a square vessel.

RULE. *Take the dimensions in inches; then multiply the length, breadth, and height together; divide the product by* 282 *for ale gallons,* 231 *for wine gallons, and* 2150.42 *for bushels.*

22. To find the contents of a cask.

RULE. *Take the dimensions of the cask in inches; viz. the diameter of the bung and head, and the length of the cask. Note the difference between the bung diameter and the head diameter. If the staves of the cask be much curved between the bung and the head, multiply the difference by .7; if not quite so much curved, by .65; if they bulge yet less, by .6; and, if they are almost straight, by .55; add the product to the head diameter; the sum will be a mean diameter by which the cask is reduced to a cylinder.*

Square the mean diameter thus found, then multiply it by the length; divide the product by 359 *for ale or beer gallons, and by* 294 *for wine gallons.*

23. To find the contents of a round vessel, wider at one end than the other.

RULE. *Multiply the greater diameter by the less; to this product, add* $\frac{1}{3}$ *of the square of their difference, then multiply by the height, and divide as in the last rule.*

24. To measure round timber.

RULE. *Multiply the length of the stick, taken in feet, by the square of* $\frac{1}{4}$ *the girt, taken in inches; divide this product by* 144, *and the quotient is the contents in cubic feet.*

NOTE. The girt is usually taken about $\frac{1}{3}$ the distance from the larger to the smaller end.

1. What are the contents of a board 25 feet long and 3 feet wide? Ans. 75 feet.
2. What is the difference between the contents of two floors; one is 37 feet long and 27 feet wide, and the other is 40 feet long and 20 feet wide? Ans. 199 feet.
3. The base of a rhombus is 15 feet, and its perpendicular height is 12 feet; what are its contents? Ans. 180 feet.
4. What are the contents of a triangle, whose base is 24 feet, and whose perpendicular height is 18 feet? Ans. 216 feet.
5. What are the contents of a triangular piece of land, whose sides are 50 rods, 60 rods, and 70 rods? Ans. 1469.69+ rods.
6. What is the circumference of a circle, whose diameter is 50 feet? Ans. 157.0796+ feet.
7. We have a round field 40 rods in diameter; what is the side of a square field, that will contain the same quantity? Ans. 35.44+ rods.
8. What is the side of an equilateral triangle, that may be inscribed in a circle 50 feet in diameter? Ans. 35.35+ feet.
9. If the diameter of a circle be 200 feet, what is the area? Ans. 31415.92+ feet.
10. What is the diameter of a circle, whose circumference is 80 miles? Ans. 25.46+ miles.
11. I have a circular field 100 rods in circumference; what must be the side of a square field, that shall contain the same area? Ans. 28.2+ rods.
12. Required the side of a triangle, that may be inscribed in a circle, whose circumference is 1000 feet. Ans. 275.66+ feet.
13. How large a square field may be inscribed in a circle, whose circumference is 100 rods? Ans. 22.5+ rods square.
14. How many cubic feet are there in a cube whose sides are 8 feet? Ans. 512 feet.
15. What is the difference between the number of *cubic* feet in a room 30 feet long, 20 feet wide, and 10 feet

high, and the number of *square* feet in the surface of the room? Ans. 6000 solid feet. 2200 square feet.

16. What are the contents of a triangular prism, whose length is 20 feet, and the three sides of its triangular end or base 5, 4, and 3 feet? Ans. 120 feet.

17. What are the solid contents of a cone, whose height is 30 feet, and the diameter of its base 5 feet?
Ans. 196.3+ feet.

18. The largest of the Egyptian pyramids is square at its base, and measures 693 feet on a side. Its height is 500 feet. Now, supposing it to come to a point at its vertex, what are its solid contents, and how many miles in length of wall would it make, 4 feet in height and 2 feet thick?
Ans. 80,041,500 cubic feet. 1894.9 miles in length.

19. Required the convex surface of a cone, whose side is 50 feet, and the circumference at its base 12 feet.
Ans. 300 feet

20. Required the solid contents of Bunker Hill monument, whose height is 220 feet, and being 30 feet square at its base, and 15 feet square at its vertex.
Ans. 115500 cubic feet.

21. What are the contents of a stick of timber 20 feet long, and the diameter at the larger end 12 inches, and at the smaller end 6 inches? Ans. 9.163+ feet.

22. What is the solidity of a sphere, whose diameter is 20 inches? Ans. 4188.8+ inches.

23. What is the convex surface of a globe, whose diameter is 20 inches? Ans. 1256.6+ inches.

24. What are the contents of a spherical segment 3 feet in height, cut from a sphere 10 feet in diameter?
Ans. 113.0976 feet.

25. What is the solidity of a segment of a sphere, its height being 8 inches, and the diameter of its base 20 inches? Ans. 1524.7232 inches.

26. How large a cube may be inscribed in a sphere 10 inches in diameter? Ans. 5.773+ inches.

27. How many wine gallons will a cubical box contain, that is 8 feet long, 4 feet high, and 3 feet wide?
Ans. 718.1+ gallons.

28. How many bushels of grain will a box contain, that is 12 feet long, 5 feet wide, and 4 feet high?
Ans. 192.8+ bushels.

29. What are the contents of a cask, in wine gallons, whose bung diameter is 30 inches, head diameter 24 inches, and length 40 inches? Ans. 108.19+ gallons.

30. How many cubic feet in a stick of timber, which is 40 feet long, and whose girt is 60 inches?

Ans. $62\frac{1}{2}$ feet.

Section 51.

MISCELLANEOUS QUESTIONS.

1. What is the difference between 7 pence and 10 cents?

Ans. $\frac{1}{5}$d.

2. What number is that, to which, if $\frac{1}{8}$ be added, the sum will be $7\frac{1}{2}$? Ans. $7\frac{3}{8}$.

3. What number is that, from which, if $3\frac{3}{7}$ be taken, the remainder will be $4\frac{1}{2}$? Ans. $7\frac{1}{2}\frac{5}{8}$.

4. What number is that, to which, if $3\frac{3}{7}$ be added, and the sum divided by $5\frac{3}{7}$, the quotient will be 5?

Ans. $23\frac{3}{7}$.

5. From $\frac{7}{11}$ of a mile take $\frac{7}{8}$ of a furlong.

Ans. 4fur. 12rd. 8ft. 8in.

6. From 7 acres take $\frac{9}{11}$ of a rood.

Ans. 6A. 3R. 7p. 74ft. 36in.

7. John Swift can travel 7 miles in $\frac{8}{9}$ of an hour, but Thomas Slow can travel only 5 miles in $\frac{7}{11}$ of an hour. Both started from Danvers at the same time for Boston, the distance being 12 miles. How much sooner will Swift arrive in Boston than Slow? Ans. $12\frac{3}{7}\frac{3}{7}$ seconds.

9. If $\frac{5}{8}$ of a ton cost $49, what cost 1cwt.?

Ans. $3.92.

9. How many bricks, 8 inches long, 4 inches wide, and 2 inches thick, will it take to build a wall 40 feet long, 20 feet high, and 2 feet thick? Ans. 43200 bricks.

10. How many bricks will it take to build the walls of a house, which is 80 feet long, 40 feet wide, and 25 feet high, the wall to be 12 inches thick; the brick being of the same dimensions, as in the last question?

Ans. 159300 bricks.

11. How many tiles, 8 inches square, will cover a floor 18 feet long, and 12 feet wide? Ans. 486 tiles.

12. If it cost $18.25 to carry 11cwt. 3qr. 19lbs. 46 miles, how much must be paid for carrying 83cwt. 2qr. 11lbs. 96 miles? Ans. 267.12\frac{456}{2047}$.

13. A merchant sold a piece of cloth for $24, and thereby lost 25 per cent.; what would he have gained, had he sold it for $34? Ans. 6$\frac{1}{8}$ per cent.

14. Bought a hogshead of molasses, containing 120 gallons, for $30; but 20 gallons having leaked out, for what must I sell the remainder per gallon to gain $10? Ans. $0.40.

15. In a piece of land 117$\frac{3}{7}$ rods long, and 112$\frac{2}{5}$ rods wide, how many acres?

Ans. 82A. 1R. 18p. 2yd. 7ft. 133$\frac{5}{7}$in.

16. Bought a quantity of goods for $128.25, and, having kept them on hand 6 months, for what must I sell them to gain 6 per cent.? Ans. $140.02.

17. If 27 bushels of potatoes cost $8.75, what must be paid for 36 bushels? Ans. $11.66+.

18. How many bushels of oats, at 50 cents per bushel, must I give Moses Webster for 93 bushels of corn, at $1.25 per bushel? Ans. 232$\frac{1}{2}$ bushels.

19. How many bushels of salt, at $1.30 per bushel, must be given in exchange for 75 bushels of wheat, at $1.25 per bushel? Ans. 72$\frac{3}{26}$ bushels.

20. If a sportsman spend $\frac{1}{3}$ of his time in smoking, $\frac{1}{4}$ in "gunning," 2 hours per day in *loafing*, and 6 hours in eating, drinking, and sleeping, how much remains for useful purposes? Ans. 2 hours.

21. If a lady spend $\frac{1}{4}$ of her time in sleep, $\frac{1}{5}$ in making *calls*, $\frac{1}{6}$ at her toilet, $\frac{1}{7}$ in reading novels, and 2 hours each day in receiving visits, how large a portion of her time will remain for improving her mind, and domestic employments? Ans. 3$\frac{17}{35}$ hours per day.

22. What will a piece of land 7$\frac{3}{5}$ rods long, and 5$\frac{4}{5}$ rods wide, come to at $25.75 per acre? Ans. 6.65\frac{133}{144}$.

23. If 5$\frac{3}{5}$ ells English cost $15.16, what will 71$\frac{1}{4}$ yards cost; Ans. $155.39.

24. If a staff 4 feet long cast a shadow 5$\frac{3}{5}$ feet, what is the height of that steeple whose shadow is 150 feet?

Ans. 107$\frac{1}{7}$ feet.

25. Borrowed of James Day $150 for six months; afterwards I lent him $100; how long shall he keep it to indemnify him for the sum he lent me? Ans. 9 months.

26. A certain town is taxed $6045.50; the valuation of the town is $293275.00; there are 150 polls in the town, which are taxed $1.20 each. What is the tax on a dollar, and what does A. pay, who has 4 polls, and whose property is valued at $3675?

Ans. $0.02. A.'s tax $78.30.

27. What is the value of 97 pigs of lead, each weighing 2cwt. 3qr. 11lb., at £3. 17s. 9d. per cwt.?

Ans. £1074. 0s. $6\frac{37}{112}$d.

28. What is the interest of $17.86, from Feb. 9, 1840, to Oct. 29, 1842, at $7\frac{1}{4}$ per cent.? Ans. $35.24+.

29. What is the interest of $97.87, from Jan. 7, 1840, to Sept. 25, 1842, at 9 per cent.? Ans. $23.92+.

30. T. Jones' note for $1728 is dated March 1, 1836;

Sept. 25, 1836, was received		$50.00,
Jan. 1, 1837,	do.	$60.00,
June 7, 1837,	do.	$8.00,
Dec. 25, 1837,	do.	$10.00,
March 6, 1838,	do.	$5.00,
Sept. 1, 1838,	do.	$9.00,
Jan. 1, 1839,	do.	$300.00,
July 4, 1839,	do.	$100.00,
Sept. 6, 1840,	do.	$14.00,
Jan. 25, 1841,	do.	$500.00,
Dec. 11, 1841,	do.	$15.00,
March 9, 1842,	do.	$200.00,

What is due Nov. 29, 1842? Ans. $1060.29.

31. $1000. Salem, N. H., Oct. 29, 1836.

For value received, I promise to pay Luther Emerson, Jr., or order, on demand, one thousand dollars with interest. Emerson Luther.

Attest, Adams Ayer.

On this note are the following indorsements.

Jan. 1, 1837, was received		$125.00,
June 5, 1837,	do.	$316 00,
Sept. 25, 1837,	do.	$417.00,
April 1, 1838,	do.	$100.00,
July 7, 1838,	do.	$50.00;

What is due, at compound interest, Oct. 29, 1842

Ans. $53.79.

32. J. Ladd's garden is 100 feet long and 80 feet wide; he wishes to enclose it with a ditch 4 feet wide; how deep must it be dug, that the soil taken from it may raise the surface one foot. Ans. $5\frac{10}{13}$ feet.

33. How many yards of paper, that is 30 inches wide, will it require to cover the walls of a room, that is $15\frac{1}{2}$ feet long, $11\frac{1}{4}$ feet wide, and $7\frac{3}{4}$ feet high?

Ans. $55\frac{17}{60}$ yards.

34. Charles Carleton has agreed to plaster the above room at 10 cents per square yard; what will be his bill? Ans. $\$6.54\frac{4}{9}$.

35. How many cubic inches are contained in a cube, that may be inscribed in a sphere 40 inches in diameter? Ans. 12316.8+ inches.

36. The dimensions of a bushel measure are $18\frac{1}{2}$ inches wide, and 8 inches deep; what should be the dimensions of a similar measure, that would contain 4 quarts?

Ans. $9\frac{1}{4}$ inches wide, 4 inches deep.

37. A gentleman willed $\frac{1}{3}$ of his estate to his wife, and $\frac{1}{4}$ of the remainder to his oldest son, and $\frac{1}{8}$ of the residue, which was $\$151.33\frac{1}{3}$, to his oldest daughter; how much of his estate is left to be divided among his other heirs?

Ans. $\$756.66\frac{2}{3}$.

38. A man bequeathed $\frac{1}{4}$ of his estate to his son, and $\frac{1}{5}$ of the remainder to his daughter, and the residue to his wife; the difference between his son and daughter's portion was $\$100$; what did he give his wife?

Ans. $\$600.00$.

39. A young man lost $\frac{1}{4}$ of his capital in speculation; he afterwards gained $\$500$; his capital then was $\$1250$; what was the sum lost? Ans. $\$250.00$.

40. From $\frac{1}{7}$ of a yard, there was sold $\frac{1}{8}$ of it; how much remained? Ans. $\frac{8}{56}$ yard.

41. Sold a lot of shingles for $\$50$, and by so doing I gained $12\frac{1}{2}$ per cent.? what was their value?

Ans. $\$44.44\frac{4}{9}$.

42. If tallow be sold at $7\frac{1}{2}$d. per lb., what is the value of 17cwt. 3qr. 18lbs.? Ans. $\$208.95\frac{5}{8}$.

43. If $\frac{8}{11}$ of a yard cost $\$5.00$, what quantity will $\$17.50$ purchase? Ans. $\frac{7}{22}$ yard.

44. If a man travel 17rd. 10ft. in $\frac{7}{17}$ of an hour, how far will he travel in $8\frac{4}{5}$ hours?

Ans. 1 mile, $928\frac{4}{5}$ feet.

45. When $11.75 are paid for $2\frac{3}{4}$ acres, what quantity will $100.00 purchase? Ans. 19A. 1R. $32\frac{152}{320}$p.

46. John Savory and Thomas Hardy traded in company; Savory put in for capital $1000; they gained $128.00; Hardy received for his share of the gains $70; what was his capital? Ans. 1206.89\frac{8}{23}$.

47. E. Fuller lent a certain sum of money to C. Lamson, and, at the end of 3 years, 7 months, and 20 days, he received interest and principal $1000; what was the sum lent? Ans. 820.79\frac{551}{731}$.

48. Lent $88 for 18 months, and received for interest and principal $97.57; what was the per cent.?

Ans. $7\frac{1}{2}$ per cent.

49. When $\frac{4}{5}$ of a gallon cost $87, what cost $7\frac{1}{2}$ gallons?

Ans. $1051.25.

50. When $71 are paid for $18\frac{3}{4}$ yards of broadcloth, what cost 5 yards? Ans. 19.26\frac{48}{125}$.

51. How many yards of cloth, at $4.00 per yard, must be given for 18tons. 17cwt. 3qr. of sugar, at $9.50 per cwt.? Ans. $897\frac{5}{32}$ yards.

52. How much grain, at $1.25 per bushel, must be given for 98 bushels of salt, at $0.45 per bushel?

Ans. $35\frac{7}{25}$ bushels.

53. How many acres of land, at $37.50 per acre must be given for 86tons. 18cwt. 3qr. 20lbs. of coal, at $8.50 per ton? Ans. 19A. 2R. $33\frac{8}{35}$p.

54. A person, being asked the time of day, replied, that $\frac{1}{4}$ of the time passed from noon was equal to $\frac{1}{11}$ of the time to midnight. Required the time.

Ans. 40 minutes past 4.

55. How many cubic feet of water in a pond, that contains 200 acres, and is 20 feet deep?

Ans. 174,240,000 feet.

56. On a certain night, in the year 1842, rain fell to the depth of 3 inches in the town of Haverhill; the town contains about 20,000 square acres. Required the number of hogsheads of water fallen, supposing each hogshead to contain 100 gallons, and each gallon 282 cubic inches. Ans. 13346042hhd. 55gal. 1qt. 0pt. $2\frac{1}{17}$gi.

57. If the sun pass over one degree in 4 minutes, and the longitude of Boston is 71° 4′ west, what will be the

time at Boston, when it is 11h. 16m. A. M. at London?
Ans. 6h. 31m. 44sec. A. M.

58. When it is 2h. 36m. A. M. at the Cape of Good Hope, in longitude 18° 24′ east, what is the time at Cape Horn, in longitude 67° 21′ west?
Ans. 8h. 53m. P. M.

59. Yesterday my longitude, at noon, was 16° 18′ west; to-day I perceive by my watch, which has kept correct time, that the sun is on the meridian at 11h. 36m.; what is my longitude? Ans. 22° 18′ west.

60. Sound, uninterrupted, will pass 1142 feet in one second, how long will it be in passing from Boston to London, the distance being about 3000 miles?
Ans. 3h. 51m. 10$\frac{1}{2}$sec.

61. The time which elapsed between seeing the flash of a gun, and hearing its report, was 10 seconds; what was the distance? Ans. 2 miles. 860 feet.

62. If a globe of silver, 2 inches in diameter, be worth $125, what would be the value of a globe 3 inches in diameter? Ans. 421.87\frac{1}{2}$.

63. J. Pearson has tea, which he barters with M. Swift, at 10 cents per lb. more than it costs him, against sugar, which costs Swift 15 cents per lb., but which he puts at 20 cents per lb., what was the first cost of the tea?
Ans. $0.30.

64. Q. and Y. barter; Q. makes of 10 cents 12$\frac{1}{2}$ cents; Y. makes of 15 cents 19 cents; who makes the most per cent., and by how much?
Ans. Y. makes 1$\frac{3}{5}$ per cent. more than Q.

65. A certain individual was born in 1786, September 25, at 27 minutes past 3 o'clock, A. M., how many minutes old will he be July 4, 1844, at 30 minutes past 5 o'clock, P. M.? Ans. 30,386,283 minutes.

66. The longitude of a certain star is 3s. 14°. 26′. 14″., and the longitude of the moon at the same time is 8s. 19°. 43′ 28″., how far will the moon have to move in her orbit to be in conjunction with the star?
Ans. 6s. 24°. 42′. 46″.

67. From a small field containing 3A. 1R. 23p. 200ft., there were sold 1A. 2R. 37p. 30yd. 8ft.; what quantity remained? Ans. 1A. 2R. 25p. 21yd. 5ft. 36in.

68. What part of $\frac{3}{4}$ of an acre is $\frac{5}{8}$ of an acre?
Ans. $\frac{20}{21}$.

69. My chaise having been injured by a very bad boy, I am obliged to sell it for $68.75, which is 40 per cent. less than its original value, what was the cost ?

Ans. 114.58\frac{1}{3}$.

70. Charles Webster's horse is valued at $120, but he will not sell him for less than $134.40 ; what per cent. does he intend to make ? Ans. 12 per cent.

71. Three merchants, L. Emerson, E. Bailey, and S. Curtiss engaged in a cotton speculation. Emerson advanced $3600, Bailey $4200 and Curtiss $2200. They invested their whole capital in cotton, for which they received $15000 in bills on a bank in New Orleans. These bills were sold to a Boston broker at 15 per cent. below par, what is each man's net gain ?

Ans. Emerson $990.00. Bailey $1155.00. Curtiss $605.00.

72. Bought a box made of a plank 3$\frac{1}{2}$ inches thick. Its length is 4ft. 9in., its breadth 3ft. 7in., and its height 2ft. 11in. How many square feet did it require to make the box, and how many cubic feet does it contain ?

Ans. 70$\frac{5}{7}$ square feet, 29$\frac{1}{2}$ cubic feet.

73. How many bricks will it require to construct the walls of a house, 64 feet long and 32 feet wide, and 28 feet high ; the walls are to be 1ft. 4in. thick, and there are also three doors 7ft. 4in. high, and 3ft. 8in. wide ; also 14 windows 3 feet wide and 6 feet high, and 16 windows 2ft. 8in. wide and 5ft. 8in. high. Each brick is to be 8 inches long, 4 inches wide, and 2 inches thick.

Ans. 167,480 bricks.

74. John Brown gave to his three sons, Benjamin, Samuel, and William, $1000 to be divided in the proportion of $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{7}$ respectively ; but William, having received a fortune by his wife, resigns his share to his brothers. It is required to divide the whole sum between Benjamin and Samuel.

Ans. Benjamin 571.42\frac{8}{7}$. Samuel 428.57\frac{1}{7}$.

75. Peter Webster rented a house for one year to Thomas Bailey for $100 ; at the end of four months, Bailey rented one half of the house to John Bricket, and at the end of eight months, it was agreed by Bricket and Bailey to rent one third of the house to John Dana. What share of the rent must each pay ?

Ans. Bailey 61\frac{1}{3}$, Bricket 27\frac{7}{3}$, and Dana 11\frac{1}{3}$.

76. Bought 365 yards of broadcloth, for which I paid £576. 17s. 9d.; for how much must the cloth be sold per yard to gain 25 per cent. Ans. £1. 19s. $6\frac{44}{292}$d.

77. John Brown's house is 40 feet square; the roof comes to a point over the centre of the house, and this point is 12 feet above the garret floor. Required the length of a rafter, which extends from one of the corners of the house to the highest part of the roof.

Ans. 30.72+ feet.

78. Minot Thayer sold broadcloth at $4.40 per yard, and by so doing he lost 12 per cent.; whereas he ought to have gained 10 per cent. For what should the cloth have been sold per yard? Ans. $5.50.

79. John Crowell sold cloth at $5.50 per yard, and gained 10 per cent.; whereas, the cloth having been damaged, he should have sold it 12 per cent. less than the cost. What *in justice* should he have charged per yard? Ans. $4.40.

80. Jacob How has cloth, which he purchased for 12 per cent. less than its value; but he sells it at 10 per cent. more than it is worth, and by so doing he gains $1.10 on each yard. What per cent. did he make on his purchase? Ans. 25 per cent.

81. A gentleman has five daughters, Emily, Jane, Betsey, Abigail, and Nancy, whose fortunes are as follows. The first two and the last two have $19,000; the first four $19,200; the last four $20,000; the first and the last three $20,500; the first three and the last $21,300. What was the fortune of each?

Ans. Emily has $5,000; Jane $4,500; Betsey $6,000; Abigail $3,700; and Nancy $5,800.

APPENDIX.

CANCELLING METHOD.

By the Cancelling Method the scholar is enabled to solve many questions with less than half the labor, that would be required by the usual process. It cannot, however, be applied to all the rules of arithmetic, nor to all the questions under any one rule; but it is generally used in the operations of those questions which require Multiplication and Division. The system is not new. It has been before the public in some form or other for centuries. John Birks, who published the second edition of his most excellent system of "Arithmetical Collections" in London, 1764, has made many improvements in the system. Since that period, but little advance has been made in it. Whether the author has made his system more plain and intelligible than has been done by others, the candid public must judge. He has spared no pains to exhibit its applicability and utility to those departments of arithmetical science where it can be advantageously employed. He believes the system can be of but little use to the pupil, until he can perform the questions by the common method. Hence the propriety of deferring attention to this method, until the common rules of arithmetic are thoroughly understood.

GENERAL RULE.

1. *Equal divisors and dividends cancel each other.*

2. *When the* product *of two divisors is equal to the* product *of two dividends, they cancel each other.*

I. Cancelling applied to Compound Fractions.

RULE 1. — *If there be numbers in the numerators and denominators, that be alike, an equal number of the same value may be cancelled.*

1. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{7}{8}$ of $\frac{8}{9}$ to a simple fraction.

STATEMENT. CANCELLED.

$$\frac{2\times3\times4\times7\times8}{3\times4\times5\times8\times9}=\frac{2\times\cancel{3}\times\cancel{4}\times7\times\cancel{8}}{\cancel{3}\times\cancel{4}\times5\times\cancel{8}\times\cancel{9}}=\frac{14}{45}\text{ Ans.}$$

In this question, we find a 3, 4, and 8 among the numerators, and also the same numbers among the denominators. These we cancel before we commence the operation.

2. What is the value of $\frac{7}{8}$ of $\frac{8}{11}$ of $\frac{11}{17}$ of $\frac{17}{19}$?

OPERATION.

$$\frac{7\times\cancel{8}\times\cancel{11}\times\cancel{17}}{\cancel{8}\times\cancel{11}\times\cancel{17}\times19}=\frac{7}{19}\text{ Ans.}$$

We find in this question, 8, 11, and 17 among the numerators, also the same numbers among the denominators. These we cancel.

3. What is the value of $\frac{7}{8}$ of $\frac{8}{13}$ of $\frac{13}{15}$ of $\frac{7}{10}$ of $\frac{15}{17}$ of \$25.

$$\frac{7\times\cancel{8}\times\cancel{13}\times7\times\cancel{15}\times25}{\cancel{8}\times\cancel{13}\times\cancel{15}\times10\times17\times1}=\frac{1225}{170}=\$7\tfrac{1}{34}\text{ Ans.}$$

4. Reduce $\frac{4}{11}$ of $\frac{11}{12}$ of $\frac{12}{17}$ of $\frac{17}{19}$ of $4\frac{3}{4}$ to a simple fraction.

$$\frac{5\times\cancel{11}\times\cancel{12}\times\cancel{17}\times\cancel{19}}{\cancel{11}\times\cancel{12}\times\cancel{17}\times\cancel{19}\times4}=\frac{5}{4}=1\tfrac{1}{4}\text{ Ans.}$$

5. Required the value of $\frac{7}{8}$ of $\frac{8}{10}$ of $\frac{10}{13}$ of $\frac{13}{24}$ of 40.

$$\frac{7\times\cancel{8}\times\cancel{10}\times\cancel{13}\times40}{\cancel{8}\times\cancel{10}\times\cancel{13}\times24}=\frac{280}{24}=11\tfrac{2}{3}\text{ Ans.}$$

6. Reduce $\frac{11}{15}$ of $\frac{15}{16}$ of $2\frac{1}{7}$ to its equivalent value.

$$\frac{11\times\cancel{15}\times\cancel{16}}{\cancel{15}\times\cancel{16}\times7}=\frac{11}{7}=1\tfrac{4}{7}\text{ Ans.}$$

7. What is the value of $\frac{5}{11}$ of $1\frac{1}{7}$ of $\frac{7}{19}$ of $3\frac{1}{5}$ of \$18?

$$\frac{\cancel{5}\times\cancel{11}\times\cancel{7}\times\cancel{19}\times18}{\cancel{11}\times\cancel{7}\times\cancel{19}\times\cancel{5}\times1}=\frac{18}{1}=\$18\text{ Ans.}$$

8. What is the value of $\frac{7}{11}$ of $\frac{11}{25}$ of $\frac{25}{31}$ of $\$7\frac{3}{4}$?

$$\frac{7 \times \cancel{11} \times \cancel{25} \times \cancel{31}}{\cancel{11} \times \cancel{25} \times \cancel{31} \times 4} = \frac{7}{4} = \$1.75 \text{ Ans.}$$

9. What is $\frac{4}{5}$ of $\frac{9}{17}$ of $\frac{17}{18}$ of $3\frac{3}{4}$ gallons?

$$\frac{4 \times \cancel{9} \times \cancel{17} \times \cancel{18}}{\cancel{9} \times \cancel{17} \times \cancel{18} \times 5} = \frac{4}{5} \text{ gal. Ans.}$$

RULE 2. — *When there are any two numbers, one in the numerators, and the other in the denominators, which may be divided by a number without a remainder, the quotients arising from such division may be used in the operation of the question, instead of the original numbers. The quotients also may be cancelled, as other numbers.*

1. Reduce $\frac{4}{7}$ of $\frac{14}{27}$ of $\frac{21}{25}$ of $\frac{5}{11}$ to its lowest terms.

OPERATION.

$$\frac{\overset{2}{\cancel{4}} \times \overset{7}{\cancel{14}} \times \overset{1}{\cancel{21}} \times \cancel{5}}{\cancel{7} \times \underset{9}{\cancel{27}} \times \underset{5}{\cancel{25}} \times 11} = \frac{56}{495} \text{ Ans.}$$

In performing this question, we find that 14 among the numerators, and 7 among the denominators, may be divided by 7, and that their quotients will be 2 and 1. We write the 2 *above* the 14, and 1 *below* the 7. We also find a 21 among the numerators, and a 27 among the denominators, which may be divided by 3, and that their quotients will be 7 and 9. We write the 7 *above* the 21, and 9 *below* the 27. We again find a 5 among the numerators, and a 25 among the denominators, which may be divided by 5, and that their quotients will be 1 and 5. We write the 1 *over* the 5, and the 5 *below* the 25. We then multiply the 4, 2, 7, and 1 together for a numerator = 56, and the 1, 9, 5, and 11 for a denominator = 495. The answer will therefore be $\frac{56}{495}$.

2. Reduce $\frac{14}{15}$ of $\frac{18}{25}$ of $\frac{10}{11}$ of $\frac{3}{21}$ to a simple fraction.

$$\frac{\overset{2}{\cancel{14}} \times \overset{6}{\cancel{18}} \times \overset{2}{\cancel{10}} \times \overset{1}{\cancel{3}}}{\underset{5}{\cancel{15}} \times \underset{5}{\cancel{25}} \times 11 \times \underset{3}{\cancel{21}}} = \frac{24}{275} \text{ Ans.}$$

3. What is the value of $\frac{1}{4}$ of $\frac{4}{5}$ of $\frac{11}{14}$ of $\frac{14}{11}$ of $34?

$$\frac{\overset{1}{\cancel{4}}\times 9\times\overset{3}{\cancel{15}}\times\overset{2}{\cancel{14}}\times\overset{2}{\cancel{24}}}{\underset{1}{\cancel{7}}\times\underset{4}{\cancel{20}}\times\underset{4}{\cancel{16}}\times\underset{1}{\cancel{17}}\times 1}=\frac{27}{4}=\$6.75 \text{ Ans.}$$

NOTE. The above rule will apply, when the product of several numbers is to be divided by the product of other numbers.

4. What is the continued product of 8, 4, 9, 2, 12, 16, and 5 divided by the continued product of 40, 6, 6, 3, 8, 4, and 20?

$$\frac{\overset{1}{\cancel{8}}\times\cancel{4}\times\cancel{9}\times\cancel{2}\times\cancel{12}\times\cancel{16}\times\cancel{5}}{\underset{5}{\cancel{40}}\times\cancel{6}\times\cancel{6}\times\cancel{3}\times\cancel{8}\times\cancel{4}\times\cancel{20}}=\frac{1}{5} \text{ Ans.}$$

The product of 4 and 9 in the *upper* line is equal to the product of 6 and 6 in the *lower*, therefore they are cancelled; and the product of 2 and 12 in the *upper* line is equal to the product of 3 and 8 in the *lower* line; also the product of 16 and 5 in the *upper* line is equal to the product of 4 and 20 in the *lower* line; these are all cancelled. We also find, that the 8 in the upper line and the 40 in the lower line may be divided by 8, and their quotients will be 1 and 5. We write the 1 *above* the 8 and the 5 *below* the 40. By the usual process, we now find our answer is $\frac{1}{5}$.

5. What is the continued product of 12, 13, 14, 15, 16, 18, 20, 21, and 24, divided by the continued product of 2, 3, 4, 5, 6, 7, 8, 9, 10, and 11?

$$\frac{\overset{3}{\cancel{12}}\times 13\times\overset{2}{\cancel{14}}\times\overset{3}{\cancel{15}}\times\overset{2}{\cancel{16}}\times\overset{2}{\cancel{18}}\times\overset{2}{\cancel{20}}\times\overset{7}{\cancel{21}}\times\overset{2}{\cancel{24}}}{\underset{1}{\cancel{2}}\times\underset{1}{\cancel{3}}\times\underset{1}{\cancel{4}}\times\underset{1}{\cancel{5}}\times\underset{1}{\cancel{6}}\times\underset{1}{\cancel{7}}\times\underset{1}{\cancel{8}}\times\underset{1}{\cancel{9}}\times\underset{1}{\cancel{10}}\times 11}=\frac{26208}{11}=2382\tfrac{6}{11}$$

[Ans.

II. In finding the common multiple of two or more numbers, any one number that will measure another may be cancelled.

1. What is the least common multiple of 4, 6, 8, 12, 16, 10, and 20?

$$\begin{array}{r|l} 4) & \cancel{4}\ \cancel{6}\ \cancel{8}\ 12\ 16\ \cancel{10}\ 20 \\ \hline & 3\ \ 4\ \ \ \ 5 \end{array} \qquad 4\times 3\times 4\times 5=240 \text{ Ans.}$$

By examining this question, we find that 8 may be divided by 4, 12 by 6, 16 by 8, and 20 by 10; therefore we cancel 4, 6, 8, and 10.

2. What is the least common multiple of 5, 15, 30, 7, 14, and 28?

2) ~~5~~ ~~15~~ 30 ~~7~~ ~~14~~ ~~28~~
15 14

$2 \times 15 \times 14 = 420$ Ans.

In this question, we find that 15 may be measured by 5, 30 by 15, 14 by 7, and 28 by 14; we therefore cancel 5, 15, 7, and 14.

3. What is the least common multiple of 1, 2, 3, 4, 5, 6, 7, 8, and 9?

2) ~~1~~ ~~2~~ ~~3~~ ~~4~~ 5 6 7 8 9
3) 5 3 7 4 9
5 1 7 4 3

$2 \times 3 \times 5 \times 7 \times 4 \times 3 = 2520$ [Ans.

4. What is the least common multiple of 9, 8, 12, 18, 24, 36, and 72?

~~9~~ ~~8~~ ~~12~~ ~~18~~ ~~24~~ ~~36~~ 72 72 Ans.

5. What is the least number that 18, 24, 36, 12, 6, 20, and 48 will measure?

4) ~~18~~ ~~24~~ 36 ~~12~~ ~~6~~ 20 48
3) 9 5 12
3 5 4

$4 \times 3 \times 3 \times 5 \times 4 = 720$ Ans.

III. SINGLE PROPORTION,

PERFORMED BY CANCELLING.

RULE. — *When the first and second terms, or the first and third terms, can be divided by any number without a remainder, their quotients may be used in the operation of the questions instead of the terms themselves.*

1. If 14cwt. of logwood cost $56, what cost 95cwt.?

OPERATION BY PROPORTION.

cwt. cwt. $
14 : 95 :: 56
56
570
475
14)5320($380 Ans.
42
112
112
0

CANCELLING.

$$\frac{95\times\cancel{56}^{4}}{\cancel{14}_{1}} = \$380 \text{ Ans.}$$

2. If 23 men, in one month, can dig a ditch 19 rods long, 8 feet wide, and 3 feet deep, how many men would it require to dig a ditch 57 rods long, 4 feet wide, and 6 feet deep, in the same time?

BY PROPORTION.

19×8×3 : 57×4×6 :: 23
8 4
152 228
3 6
456 1368
23
4104
2736
456)31464(69 men, Ans.
2736
4104
4104

CANCELLING.

$$\frac{\cancel{57}^{3}\times\cancel{4}^{1}\times\cancel{6}^{2}\times 23}{\cancel{19}_{1}\times\cancel{8}_{2}\times\cancel{3}_{1}} = 69 \text{ men,}$$ [Ans.

3. If 7 pairs of shoes will purchase 2 pairs of boots, how many pairs of boots may be purchased with 49 pairs of shoes?

$$\frac{\cancel{49}^{7}\times 2}{\cancel{7}_{1}} = 14 \text{ pairs,}$$ [Ans.

4. If a staff 4 feet in length cast a shadow 6 feet long, how high is that steeple whose shadow is 144 feet?

$$\frac{4\times\cancel{144}^{24}}{\cancel{6}_{1}} = 96 \text{ feet,}$$ [Ans.

5. If 4 gallons of vinegar be worth 9 gallons of cider, how many gallons of cider will it require to purchase 36 gallons of vinegar?

$$\frac{\overset{9}{\cancel{36}\times 9}}{\underset{1}{\cancel{4}}} = 81 \text{ gallons, [Ans.}$$

6. If a man travel 765 miles in 75 days, how far would he travel in 15 days?

$$\frac{\overset{1}{\cancel{15}}\times\overset{153}{\cancel{765}}}{\underset{1}{\cancel{75}}} = 153 \text{ miles, [Ans.}$$

7. If 15 yards of cloth, that is 3 quarters of a yard wide, are sufficient to make a garment, how many yards will it require to line the same that is 5 quarters of a yard wide?

$$\frac{\overset{3}{\cancel{15}}\times 3}{\underset{1}{\cancel{5}}} = 9 \text{ yards, [Ans.}$$

8. When $200.85 are paid for 39 barrels of flour, what must be paid for 13 barrels?

$$\frac{\overset{66.95}{\cancel{200.85}}\times\overset{1}{\cancel{13}}}{\underset{3}{\cancel{39}}} = \$66.95 \text{ [Ans.}$$

IV. COMPOUND PROPORTION.

PERFORMED BY CANCELLING.

1. If a man travel 117 miles in 30 days, employing only 9 hours a day, how far would he go in 20 days, travelling 12 hours a day?

OPERATION.

$$\begin{array}{r|ll} \cancel{30} & \cancel{20} & \\ \cancel{9} & \cancel{12} & 4 \\ & \cancel{117} & 13 \\ \hline & 104 \text{ miles, Ans.} & \end{array}$$

In performing this question, we arrange the numbers, that would be the second and third terms in the regular statement of the question on the right hand of a perpendicular line, and the numbers, that would be the first term, on the left. We then divide the product of the uncancelled numbers on the right by the product of the uncancelled numbers on the left.

2. If 6 men in 16 days of 9 hours each build a wall 20 feet long, 6 feet high, and 4 feet thick, in how many days of 8 hours each will 24 men build a wall 200 feet long, 8 feet high, and 6 feet thick?

24 | 6
8 | 9
20 200 | 10
6 | 8
4 | 6
16

90 days, Ans.

3. If $ 100 gain $ 6 in 12 months, how much would $ 800 gain in 8 months?

100 800 | 8
12 | 8 4
6

$ 32 Ans.

4. If $ 100 gain $ 6 in 12 months, what must be the sum to gain $ 16 in 8 months?

6 | 16 2
8 | 12 2
100

$ 400 Ans.

5. How long will it take $ 600 to gain $ 12, if $ 100 gain $ 6 in 12 months?

600 100
6 | 12 2
12 2

4 months, Ans

6 If $ 600 gain $ 18 in 6 months, what is the rate per cent.?

600 100
6 | 12 2
18 3

6 per cent. Ans.

7. If 12 men in 15 days can build a wall 30 feet long, 6 feet high, and 3 feet thick, when the days are 12 hours long, in what time will 60 men build a wall 300 feet long, 8 feet high, and 6 feet thick, when they work only 8 hours a day?

5 60 | 12
30 300 | 10
6 | 8
3 | 6
8 | 12
15

120 days, Ans.

8. If 8 men spend $ 32 in 13 weeks, what will 24 men spend in 52 weeks?

8 | 24 3
13 52 | 4
32

$ 384 Ans.

9. If 16 horses consume 84 bushels of grain in 24 days, how many bushels will suffice 32 horses 48 days?

16 | 32 2
24 | 48 2
84

336 bushels, Ans.

10. If the carriage of 5cwt. 3qr., 150 miles cost $ 24.58, what must be paid for the carriage of 7cwt. 2qr. 25lbs., 64 miles at the same rate?

161 | 44 | 805 173
30 | 150 | 64 16
24.58

68037.44 / 4830 = $ 14.08+ [Ans.

11. If 7oz. 5dwt. of bread be bought at 4½d., when corn is 4s. 2d. per bushel, what weight of it may be bought for 1s. 2d., when the price per bushel is 5s. 6d.?

33 | 66 | 50 2
.19 | 4.75 | 14 7
145

2030 / 6.27 = 11lb. 4oz. 3$\frac{27}{33}$dwt. [Ans.

V. CANCELLING APPLIED TO THE CHAIN RULE.

The Chain Rule consists in joining many proportions together; and by the relations which the several antecedents have to their consequents, the proportion between the first antecedent and the last consequent is discovered.

This rule may often be abridged by cancelling equal quantities on both sides, and abbreviating commensurables.

NOTE. The first numbers in each part of the question are called *antecedents*, and the following, *consequents*.

1. If 20 lbs. at Boston make 23 lbs. at Antwerp, and 150 lbs. at Antwerp make 180 lbs. at Leghorn, how many pounds at Boston are equal to 144 lbs. at Leghorn?

OPERATION BY THE CHAIN RULE.

20 lbs. of Boston = 23 Antwerp,
150 lbs. of Antwerp = 180 Leghorn,
144 lbs. of Leghorn.

180 144
23 155
540 720
360 720
4140 144
22320
20
4140)446400(107$\frac{19}{23}$ lbs. Ans.
4140
32400
28980
3420

180) $\frac{3420}{4140} = \frac{19}{23}$

It will be perceived in this operation, that the continued product of the antecedents is divided by the continued product of the consequents.

Hence the following

RULE.—*Write the numbers alternately, that is, the antecedents at the left hand, and the consequents at the right hand; and, if the last number stands at the left hand, multiply the numbers of the left hand column continually together for a dividend, and those at the right hand for a divisor; but, if the last number stands at the right hand, multiply the numbers at the right hand column continually together for a dividend, and those at the left for a divisor; and the quotient will be the answer.*

OPERATION BY CANCELLING.

23 | 20
20 180 | 155
144 16
2480

$\frac{2480}{23} = 107\frac{19}{23}$ lbs. Ans.

2. If 12 lbs. at Boston make 10 lbs. at Amsterdam, and 10 lbs. at Amsterdam make 12 lbs. at Paris, how many pounds at Boston are equal to 80 lbs. at Paris?

10 | 12
12 | 10
80

80 lbs. Ans.

3. If 25 lbs. at Boston are equal to 22 lbs. at Nuremburg, and 88 lbs. at Nuremburg are equal to 92 lbs. at Hamburg, and 46 lbs. at Hamburg are equal to 49 lbs. at Lyons, how many pounds are equal to 98 lbs. at Lyons?

22 | 25
2 92 | 88 4
49 | 46
98 2

100 lbs. Ans.

4. If 24 shillings in Massachusetts are equal to 32 shillings in New York; and if 48 shillings in New York are equal to 45 shillings in Pennsylvania; and if 15 shillings in Pennsylvania are equal to 10 shillings in Canada; how many shillings in Canada are equal to 100 shillings in Massachusetts?

2 ~~24~~ | ~~32~~ 4
6 ~~48~~ | ~~45~~ 3
~~15~~ | 10
100
1000
$\frac{1000}{12} = 83\frac{1}{3}$s. Ans.

5. If 17 men can do as much work as 25 women, and 5 women do as much as 7 boys, how many men would it take to do the work of 75 boys?

~~25~~ | 17
7 | 5
~~75~~ 3
255
$\frac{255}{7} = 36\frac{3}{7}$ men, Ans.

6. If 10 barrels of cider will pay for 5 cords of wood, and 20 cords of wood for 4 tons of hay, how many barrels of cider will it take to purchase 50 tons of hay?

~~5~~ | ~~10~~ 2
~~4~~ | ~~20~~ 5
50
500 bls. Ans.

7. If 100 acres in Bradford be worth 120 in Haverhill, and 50 in Haverhill worth 65 in Methuen, how many acres in Bradford are equal to 150 in Methuen?

~~6~~ ~~120~~ | ~~100~~ 5
13 ~~65~~ | ~~50~~ 10
~~150~~ 25
1250
$\frac{1250}{13} = 96\frac{2}{13}$ acres, Ans.

8. If 10 lbs. of cheese are equal in value to 7 lbs. of butter, and 11 lbs. of butter to 2 bushels of corn, and 11 bushels of corn to 8 bushels of rye, and 4 bushels of rye to one cord of wood, how many pounds of cheese are equal in value to 10 cords of wood?

7 | ~~10~~ 5
~~2~~ | 11
~~2~~ ~~8~~ | 11
1 | ~~4~~
~~10~~ 5
3025
$\frac{3025}{7} = 432\frac{1}{7}$ lbs. Ans.

MISCELLANEOUS QUESTIONS.

1. Required the number of cubic feet in a box, $2\frac{1}{4}$ feet wide, $1\frac{1}{3}$ feet high, and $14\frac{7}{11}$ feet long?

$2\frac{1}{4} = \frac{9}{4}$; $1\frac{1}{3} = \frac{4}{3}$; $14\frac{7}{11} = \frac{161}{11}$.

$$\frac{\cancel{9} \times \cancel{16} \times 231}{\cancel{4} \times \cancel{9} \times \cancel{16}} = \frac{231}{4} = 57\frac{3}{4} \text{ feet, Ans.}$$

2. What cost $15\frac{3}{8}$ yards of cloth, $2\frac{2}{3}$ yards wide, at \$ $3\frac{1}{3}$ per square yard?

$15\frac{3}{8} = \frac{123}{8}$; $2\frac{2}{3} = \frac{8}{3}$; $3\frac{1}{3} = \frac{10}{3}$.

$$\frac{\overset{41}{\cancel{123} \times \cancel{8} \times 10}}{\cancel{8} \times \cancel{3} \times 3} = \frac{410}{3} = \$\ 136\frac{2}{3} \text{ Ans.}$$
$$\underset{1}{}$$

3. If \$ $12\frac{1}{2}$ will purchase a piece of land that is $9\frac{1}{4}$ rods long and $6\frac{1}{4}$ rods wide, how long a piece that is $3\frac{1}{7}$ rods wide may be obtained for \$ $9\frac{1}{2}$?

$12\frac{1}{2} = \frac{25}{2}$; $9\frac{1}{4} = \frac{37}{4}$; $6\frac{1}{4} = \frac{25}{4}$; $3\frac{1}{7} = \frac{22}{7}$; $9\frac{1}{2} = \frac{19}{2}$.

$$\frac{3 \times \cancel{37} \times \cancel{25} \times \cancel{7} \times \overset{4}{\cancel{64}}}{\cancel{37} \times \cancel{4} \times \cancel{4} \times \cancel{25} \times \cancel{7}} = 12 \text{ rods, Ans.}$$

4. When $18\frac{2}{7}$ square rods of land are sold for \$ $3\frac{1}{14}$, what is the value of $62\frac{1}{2}$ square rods?

$18\frac{2}{7} = \frac{128}{7}$; $3\frac{1}{14} = \frac{43}{14}$; $62\frac{1}{2} = \frac{125}{2}$.

$$\frac{\overset{1}{\cancel{7}} \times 125 \times \overset{1}{\cancel{43}}}{\cancel{128} \times 2 \times \cancel{14}} = \frac{125}{12} = \$\ 10\frac{5}{12} \text{ Ans.}$$
$$\underset{3}{} \qquad \underset{2}{}$$

5. How many boxes that are 1 foot 7 inches high, 1 foot 5 inches wide, and 5 feet 1 inch long, will it require to hold the same quantity that a box 4 feet 9 inches wide, 2 feet 10 inches high, and 25 feet 5 inches long, would contain?

$$\frac{\overset{3}{\cancel{57}} \times \overset{2}{\cancel{34}} \times \overset{5}{\cancel{305}}}{\cancel{19} \times \cancel{17} \times \cancel{61}} = \frac{30}{1} = 30 \text{ boxes, Ans.}$$
$$\underset{1}{} \quad \underset{1}{} \quad \underset{1}{}$$

SUPPLEMENT.

The answers to the following questions will be found in a Key to the whole work, published for teachers *only*.

Section 1.

ADDITION.

1. What is the sum of the following numbers, 87, 96, 179, 8751, 81, 410, 8117, 8, and 47876?
2. Add 1001, 76, 10078, 15, 8761, 7, and 1678.
3. Add 49, 761, 3756, 8, 150, 761761, and 18.
4. Required the sum of 3717, 8, 7, 10001, 58, 18, and 5.
5. Add 19, 181, 5, 897156, 81, 800, and 71512.
6. What is the sum of 999, 8081, 9, 1567, 88, 91, 7, and 878?
7. Add 71, 18765, 9111, 1471, 678, 9, 1446, and 71.
8. Add 51, 1, 7671, 89, 871787, 61, and 70001.
9. What is the sum of 71, 8956, 1, 785, 587, and 76178?
10. Add 9999, 8068, 8, 81, 4777, and 516785.
11. Add 5, 7, 8911, 467, 47895, and 87.
12. Add 123456, 71, 8005, 21, and 716787.
13. Add 47, 911111, 717, 81, 88767, and 56.
14. What is the sum of 71, 8899, 4, 7111, and 678679?
15. Add 81, 879, 41, 76789, 42, 1, and 78967.
16. Add 917656, 75, 876789, 46, and 6222.
17. Add 91, 76756895, 76, 14, 3, and 76378.

18. Add 10, 100, 1000, 10000, 100000, and 1000000.
19. What is the sum of 9, 99, 999, 1111, 8000, and 5?
20. Add 41, 7651, 7678956, 43, 15, and 6780.
21. Add 1234, 7891, 3146751, 27, 9, and 5.
22. What is the sum of 19, 91, 1, 1, 1478, 1007, and 46?
23. What is the sum of 489, 100, 7, 6, 5, and 1000000?
24. Add 911786, 81, 46, 71, 14, 71, 4, 10, 5, 1, and 7895.
25. Add 1, 1, 5678, 950000, 476, 13, 12, 13, and 1.
26. Add 21, 31, 41, 51, 61, 71, 18, 19, 17, 12, and 1000.
27. What is the sum of 67176, 91234, and 78956789?
28. Add 91, 787, 711178, 91, 5, 67891, 1, and 1417.
29. Add 46, 98, 881, 8, 15, 7878, 95, and 347.
30. Add 91, 8, 7678, 51, 8755, 517, and 81.
31. Add 7, 8, 9, 10, 11, 76789, 278, and 46.
32. What is the sum of 671, 81, 6789, 61, and 4444767?
33. Add 96, 9678, 9, 8, 81756, 78, and 96999.
34. Add 41, 7675, 15, 51, 789, 3, 1, 78, and 57.
35. Add 178957, 41, 717, 1, 23, 445, and 7890.
36. Add 78956, 7895, 798, 47, 4, and 99.
37. Add 41, 89, 91, 87, 15, 57, 81, 62, 71, and 83.
38. Add four hundred seventy-six, seventy-one, one hundred five, three hundred eighty-seven.
39. Add fifty-six thousand seven hundred eighty-five, seven hundred five, thirty-six, one hundred seventy thousand and one, and four hundred seven.
40. Add fifty-six thousand seven hundred eleven, three thousand seventy-one, four hundred seventy-one, sixty-one, and three thousand and one.
41. What is the sum of the following numbers, seven hundred thousand seven hundred one, seventeen thousand nine, one million six hundred thousand seven hundred six, forty-seven thousand six hundred seventy-one, seven thousand forty-seven, four hundred one, and nine?
42. Add the following numbers, six hundred ten, sixty-

seven thousand one hundred thirteen, four thousand four, seven hundred sixty thousand one hundred seventeen, and one hundred thousand seventy.

43. Add the following numbers, seventy-four thousand thirteen, nineteen thousand seven hundred eight, seven hundred sixteen, one million seven hundred eighteen thousand one, sixty thousand seven hundred eight.

44. Add eighty-seven, nine hundred fifteen, eighteen thousand, one hundred eighty, eighteen, one hundred sixty-one thousand, six hundred sixteen, eight million six hundred eight thousand, six hundred eighty-six.

Section 2.

SUBTRACTION.

1. From 715 take 103.
2. From 471 take 183.
3. From 471 take 177.
4. From 471 take 190.
5. From 471 take 197.
6. From 314 take 119.
7. From 441 take 175.
8. Take 17777 from 72341.
9. Take 17717 from 71111.
10. Take 16666 from 44444.
11. Take 18888 from 66666.
12. Take 9171 from 12345.
13. From 671111 take 199999.
14. From 1789100 take 808088.
15. From 1000000 take 999999.
16. From 9999999 take 1607.

17. From 6101507601061 take 3806790889.
18. From 8054010657811 take 76909748598.
19. From 7100071641115 take 10071178.
20. From 501505010678 take 794090589.
21. Take 99999999 from 100000000.
22. Take 44444444 from 500000000.
23. Take 1234567890 from 9987654321.
24. From 800700567 take 1010101.
25. Take twenty-five thousand twenty-five from twenty-five millions.
26. Take nine thousand ninety-nine from ninety-nine thousand.
27. From one hundred one million ten thousand one hundred one take ten million one hundred one thousand ten.
28. From one million take nine.
29. From three thousand take thirty-three.
30. From one hundred million take five thousand.

Section 3.

MULTIPLICATION.

1. Multiply 675 by 476; 471 by 395.
3. Multiply 679 by 763; 123 by 345.
5. Multiply 899 by 981; 475 by 389.
7. Multiply 7854 by 1234; 4567 by 7895.
9. Multiply 3001 by 6071; 5671 by 4316.
11. Multiply 7117 by 9876; 767123 by 100780.
13. Multiply 376546 by 407091; 78956717 by 70080016.
15. Multiply 7001009 by 7007867.
16. Multiply 8010700 by 9000909.
17. Multiply 70111111 by 11001117.
18. Multiply 10230017 by 99900099.
19. Multiply 12345678 by 89123456.

20. Multiply 99999999 by 88888888.
21. Multiply 700110000 by 700110000.
22. Multiply 4070607 by 7007000.
23. Multiply 4110000 by 1017010.
24. Multiply 1000001 by 1000001.
25. Multiply 1152921504606846976 by 1152921504606-846976.
26. Multiply five hundred eighty-six by nine hundred eighty.
27. Multiply three thousand eight hundred five by one thousand seven.
28. Multiply two thousand seventy-one by seven hundred six.
29. Multiply eighty-eight thousand and eight by three thousand and seven.
30. Multiply ninety thousand eight hundred seven by one thousand ninety-one.
31. Multiply three hundred seventy-five thousand by five thousand seven.
32. Multiply ninety thousand eight hundred seven by nine thousand one hundred six.
33. Multiply fifty thousand and one by five thousand eight hundred seven.
34. Multiply eighty thousand and nine by nine thousand sixteen.
35. Multiply forty-seven thousand thirteen by eighty thousand eight hundred seven.
36. Multiply twenty-nine million two thousand nine hundred nine by four hundred four thousand forty.
37. Multiply eighty-seven millions by eight hundred thousand seven hundred.
38. Multiply one million one thousand one hundred one by nine hundred nine thousand ninety.
39. Multiply forty-nine millions forty-nine by four hundred ninety thousand forty-nine.
40. Multiply two hundred millions two hundred by two millions two thousand and two.

Section 4.

DIVISION.

1. Divide 1728 by 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12.
11. Divide 123456789 by 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12.
21. Divide 987654 by 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12.
31. Divide 2345678901 by 23, 24, 25, 26, 27, 28, 29, 33, 35, and 37.
41. Divide 6789567 by 81, 76, 59, 97, 86, 43, and 69.
48. Divide 8901239 by 89, 46, 38, 15, 19, 91, and 99.
55. Divide 5678901 by 79, 31, 39, 47, 51, 19, and 44.
62. Divide 2345678 by 91, 18, 81, 71, 61, 51, and 41.
69. Divide 9012345 by 31, 41, 51, 61, 72, 82, and 99.
76. Divide 6717890 by 98, 88, 78, 68, 58, 48, and 38.
83. Divide 4567890 by 19, 29, 39, 49, 59, 69, and 79.
90. Divide 1357901 by 87, 77, 67, 57, 37, 47, and 27.
97. Divide 9988891 by 77, 19, 17, 16, 15, 29, and 99.
104. Divide 9999999 by 69, 41, 34, 64, 54, 84, and 94.
111. Divide 3456789567 by 987, 849, 319, and 819.
115. Divide 8997744444 by 345, 678, 907, and 419.
119. Divide 4500700701 by 407, 506, 781, and 309.
123. Divide 7010781009 by 800, 700, 601, and 123.
127. Divide 345678901765 by 4007, 671, and 39.
130. Divide 478956785178 by 56789 and 475.
132. Divide 678957000107 by 10789561.
133. Divide 990070171009 by 900700601.
134. Divide 444440000700 by 87670000.
135. Divide 987654786 by 9867.
136. Divide 985746856 by 9867.
137. Divide 9876534444444 by 4444444.
138. Divide 777777777777 by 3333333.
139. Divide 9900990000999900 by 88001.

140. Divide 800090001003007 by 9000009.

141. Divide 34567891234567 87 by 990000.

142. Divide 8000000000000000 by 777777.

143. Divide 1357924680192876 by 34567891231.

144. Divide 4766666000000 by 55550000000.

145. Divide 987654321123 by 100000000.

146. Divide 71897654325 by 700000000.

147. Divide 102340050070 by 102340050070.

148. Divide three hundred twenty-one thousand three hundred dollars equally among six hundred seventy-five men.

149. Four hundred seventy-one men purchase a township containing one hundred eighty-six thousand forty-five acres; what is the share of each?

150. A rail-road, which cost five hundred eighteen thousand seventy-seven dollars, is divided into six hundred seventy-nine shares; what is the value of each share?

151. Divide forty-two thousand four hundred thirty-five bushels of wheat equally among one hundred twenty-three men.

152. A prize, valued at one hundred eighty-four thousand seven hundred seventy-five dollars, is to be divided equally among four hundred seventy-five men; what is the share of each?

153. A certain company purchased a valuable township for nine million six hundred ninety-one thousand eight hundred thirty-six dollars; each share was valued at seven thousand eight hundred fifty-four dollars; of how many men did the company consist?

154. A tax of thirty million fifty-six thousand four hundred sixty-five dollars is assessed equally on four thousand five hundred ninety-seven towns; what sum must each town pay?

Section 5.

MISCELLANEOUS EXAMPLES.

1. A. bought 73 hogshead of molasses at 29 dollars per hogshead, and sold it at 37 dollars per hogshead; what did he gain?
2. B. bought 896 acres of wild land at 15 dollars per acre, and sold it at 43 dollars per acre; what did he gain?
3. N. Gage sold 47 bushels of corn at 57 cents per bushel, which cost him only 37 cents per bushel; how many cents did he gain?
4. A butcher bought a lot of beef weighing 765 pounds at 11 cents per pound, and sold it at 9 cents per pound; how many cents did he lose?
5. A taverner bought 29 loads of hay at 17 dollars per load, and 76 cords of wood at 5 dollars a cord; what was the amount of the hay and the wood?
6. Bought 17 yards of cotton at 15 cents per yard, 46 gallons of molasses at 28 cents per gallon, 16 pounds of tea at 76 cents a pound, and 107 pounds of coffee at 14 cents a pound; what was the amount of my bill?
7. A man travelled 78 days, and each day he walked 27 miles; what was the length of his journey?
8. A man sets out from Boston to travel to New York, the distance being 223 miles, and walks 27 miles a day for 6 days in succession; what distance remains to be travelled?
9. What cost a farm of 365 acres at 97 dollars per acre?
10. Bought 376 oxen at 36 dollars per ox, 169 cows at 27 dollars each, 765 sheep at 4 dollars per head, and 79 elegant horses at 275 dollars each; what was paid for all?
11. J. Barker has a fine orchard, consisting of 365 trees, and each tree produces 7 barrels of apples, and these apples will bring him in market 3 dollars per barrel; what is the income of the orchard?

12. J. Peabody bought of E. Ames 7 yards of his best broadcloth at 9 dollars per yard, and in payment he gave Ames a one hundred dollar bill; how many dollars must Ames return to Peabody?

13. Bought of P. Parker a cooking-stove for 31 dollars, 7 quintals of his best fish at 6 dollars per quintal, 14 bushels of rye at one dollar per bushel, and 5 mill-saws at 16 dollars each; in part payment for the above articles, I sold him 8 thousand feet of boards at 15 dollars per thousand; how much must I pay him to balance the account?

14. In one day there are 24 hours; how many in 57 days?

15. In one pound avoirdupois weight there are 16 ounces; how many ounces are there in 369 pounds?

16. In a square mile there are 640 acres; how many acres are there in a town, which contains 89 square miles?

17. What cost 78 barrels of apples at 3 dollars per barrel?

18. Bought 500 bushels of flour at 5 dollars per barrel, 47 hundred weight of cheese at 9 dollars per hundred weight, and 15 barrels of salmon at 17 dollars per barrel; what was the amount of my purchase?

19. Bought 760 acres of land at 47 dollars per acre, and sold J. Emery 171 acres at 56 dollars per acre, J. Smith 275 acres at 37 dollars per acre, and the remainder I sold to J. Kimball at 75 dollars per acre; what did I gain by my sales?

20. Bought a hogshead of oil containing 184 gallons at 75 cents per gallon; but 28 gallons having leaked out, I sold the remainder at 98 cents per gallon; did I gain or lose by my bargain?

21. Bought a quantity of flour, for which I gave 1728 dollars, there being 288 barrels; I sold the same at 8 dollars per barrel; what did I gain?

22. Purchased a cargo of molasses for 9212 dollars, there being 196 hogsheads; I sold the same at 67 dollars per hogshead; what did I gain?

Section 6.

INTEREST.

1. What is the interest of $144 for one year at 7 per cent.?

2. What is the interest of $850 for 1 year 7 months 18 days at 7 per cent.?

NOTE. When the interest required is for months and days at 7 per cent., the better way is to find the interest of the principal at 6 per cent. by rule, (pages 120 and 121) and to this sum add $\frac{1}{6}$ of the interest. Hence the preceding question should be performed in the following manner.

$850
.098
6800
7650
6) $83.300 interest at 6 per cent.
13883
$97.18,3 interest at 7 per cent.

3. What is the interest of $865.75 for 3 years 9 months 24 days at 7 per cent.?

4. What is the interest of $960.18 for 1 year 2 months at 7 per cent.?

5. What is the interest of $1728.19 for 3 years 8 months 10 days at 7 per cent.?

6. What is the interest of $17.90 for 8 months 4 days at 7 per cent.?

7. What is the interest of $1165.50 for 5 years 3 months 9 days at 7 per cent.?

8. What is the interest of $1237.90 for 1 year 7 months 3 days at 7 per cent.?

9. What is the interest of $156.80 for 3 years and 3 days at 7 per cent.?

10. What is the interest of $579.75 for 1 year 2 months 2 days at 7 per cent.?

11. What is the interest of $ 7671.09 for 2 years 8 months 5 days at 7 per cent.?

12. What is the interest of $ 943.11 for 1 month 29 days at 7 per cent.?

13. What is the interest of $ 975.06 for 2 years 7 months 9 days at 8¼ per cent.?

NOTE. See note, page 121.

14. What is the interest of $ 1371.15 for 3 years 6 months 10 days at 11¼ per cent.?

15. What is the interest of $ 871.75 for 1 year 11 months 9 days at 8¼ per cent.?

16. What is the amount of $ 976.25 for 3 years 1 month 11 days at 12½ per cent.?

17. What is the amount of $ 1000 for 3 years 3 months 29 days at 5½ per cent.?

18. What is the interest of $ 765 for two years 9 months at 1 per cent.?

19. What is the interest of $ 979.15 for 3 years 2 months 4 days at 2½ per cent.?

20. What is the interest of $ 760.75 from June 7, 1841, to March 9, 1843, at 7 per cent.?

NOTE. See rule, page 122.

21. What is the amount of $ 175.08 from May 7, 1841, to September 25, 1843, at 7 per cent.?

22. What is the amount of $ 160 from December 11, 1843, to September 9, 1844, at 7 per cent.?

23. What is the interest of $ 857.16 from February 26, 1841, to July 4, 1843, at 7¼ per cent.?

24. What is the interest of $ 171.18 from March 15, 1842, to July 17, 1844, at 9 per cent.?

25. What is the interest of $ 97.19 from August 17, 1843, to November 9, 1844, at 7 per cent.?

26. What is the amount of $ 765.75 from December 19, 1840, to October 11, 1843, at 6 per cent.?

27. What is the amount of $850 from March 19, 1843, to December 11, 1845, at 9½ per cent.?

28. What is the amount of $769.87 from April 2, 1841, to January 19, 1843, at 7 per cent.?

29. What is the interest of $1728.28 from June 7, 1842, to July 4, 1844, at 8 per cent.?

30. What is the amount of $565.25 from February 17, 1843, to January 1, 1845, at 8 per cent.?

31. $960. Newark, N. J., Oct. 23, 1840.

On demand, I promise to pay S. S. St. John, or order, nine hundred sixty dollars, for value received, with interest at seven per cent. John Q. Smith.

Attest, H. F. Wilcox.

On this note are the following indorsements.

Sept. 25, 1841. Received one hundred forty dollars.
July 7, 1842. Received eighty dollars.
Dec. 9, 1842. Received seventy dollars.
Nov. 8, 1843. Received one hundred dollars.

What is due Oct. 23, 1844?

32. $1000. New York, January 1, 1839.

Two months after date, I promise to pay S. Durand, or order, one thousand dollars, for value received, with interest after, at seven per cent.

Paul Sampson, Jr.

Attest, William S. Hall.

On this note are the following indorsements.

March 1, 1840. Received one hundred dollars.
Sept. 25, 1841. Received two hundred dollars.
Oct. 9, 1842. Received one hundred fifty dollars.
July 4, 1843. Received twenty dollars.
Oct. 9, 1843. Received three hundred dollars.

What is due Dec. 1, 1844?

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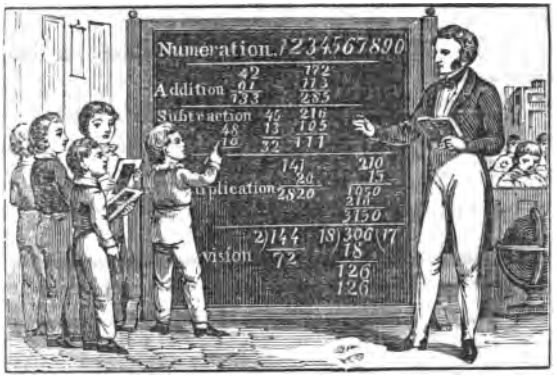

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3. *The language is simple, clear, precise and accurate, and the treatise is comprehensive, embracing a large amount of mercantile information not usually included in works of this kind, but important to be possessed by all who are destined for the warehouse or the counting-room.*

4. *The rules, the definitions, and the illustrations, are expressed in terms perfectly intelligible to the pupil; and the examples are of a practical nature, tending to interest the student, as well as to exercise his ingenuity.*

5. *The work contains a greater amount and variety of matter, strictly connected with the science of numbers, than will be found in any other treatise of the kind.*

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Instructor Natural Philosophy and Mathematics, Washington Institute.

New York, Jan'y 24, 1844.

Having had occasion to examine the several treatises upon Arithmetic used in our best schools, and to select from them one for the use of the pupils of the Commercial Department of the University Grammar School, I found Emerson's 3d part and Greenleaf's National Arithmetic to possess apparently about equal degrees of merit. After having taken several large classes through Emerson's, and several others through Greenleaf's, I have adopted the latter as the permanent Arithmetic of my school. ISAAC G. HUBBS,

New York, April 9, 1844. *Commercial and Collegiate School.*

After a careful and rigid examination of Greenleaf's Arithmetic, I do not hesitate to say that it is the *very best* work of the kind extant. I have introduced it into my school, and am delighted with the apparent satisfaction displayed by my pupils. W. KING,

New York, Oct. 2, 1843. *Classical and English School, 639 Broadway.*

I have carefully examined Greenleaf's Introduction and National Arithmetic. They are, in my opinion, better adapted to the purposes of teaching, than any other books in this department of science with which I am acquainted. I prefer them to Emerson's series which I have used during the last eight years, and I have therefore introduced them into my school.

CHAS. W. FEEKS, 649 Broadway, N. Y.,

New York, Oct. 4, 1843. *Classical and English School.*

I have examined with great care and attention "Greenleaf's National Arithmetic," and have no hesitation in saying, that I think it excels every other work of the kind with which I am acquainted. I have accordingly introduced it into "All Saints' Parochial School," and most cheerfully recommend it to others of my profession. WM. A. TAYLOR,

New York, Oct. 31, 1843. *Principal of All Saints' Parochial School.*

B. Greenleaf, Esq. Dear Sir: I thank you most heartily for the "National Arithmetic" you presented me some time ago. After a thorough and practical examination of the work, I can truly say it pleases me more than any I have ever used. The youngest scholars are interested, and love to study it; the oldest give evidence of a clearer and better understanding of the subject than they have ever shown before. Having introduced it into my own school, I would cheerfully recommend it to others. Yours, very truly,

J. JAY GREENOUGH.

New York, Sept. 12, 1843.

I have examined Mr. Greenleaf's National Arithmetic, and am so well satisfied of its superiority to any other Arithmetic with which I am acquainted, that I shall introduce it into my school at the earliest opportunity.

JEREMIAH BAKER,
Teacher of the Juvenile Department of the Cornelius Institute.
New York, Jan. 5, 1844.

I have examined, with some care, Mr. Greenleaf's Arithmetic; and, by that examination, imperfect as it was, am forced to the conclusion, that, on several accounts, it is preferable to most, if not all, other works on that subject, that I have seen.

JAMES N. MCELLIGOTT,
New York, Jan., 1844. *Principal of the Mechanic Society School.*

This is to certify, that in this school we have recently introduced the National Arithmetic, by Benjamin Greenleaf, and have used it much to our satisfaction. We have been even more pleased than an inspection of the work led us to hope. It is brief and practical in its method; and its rules and illustrations are, in my opinion, of the first order.

J. MORRISON REID, A. M.,
New York, April, 1844. *Principal of Mechanics' Institute School.*

I have examined with much attention Greenleaf's National Arithmetic and the Introduction to the same, and consider them superior to any other Arithmetic with which I am acquainted. The execution of the works is also highly to be commended. I have introduced them into my school.

J. B. QUICK,
New York, Sept. 13, 1843. *Principal City Commercial School.*

GREENLEAF'S ARITHMETIC. — This work, whether considered in regard to originality of system, diversity of rules, variety of new and useful problems, and perspicuously illustrated examples; or in regard to its complete adaptation to the use of schools, and to the future profession of the scholar, either as merchant, mechanic, farmer, or public officer, surpasses, in my opinion, all that I have examined.

The numerous exercises in analysis, cancelling, and mental calculations, cannot fail to mature the judgment, and greatly accelerate the progress of the student. Having used it during three months in my school, I have finally adopted it, firmly believing that a better knowledge of arithmetic can be obtained from it than from any other.

E. L. AVERY,
New York, March 14, 1844. *Principal of Academy, 67 Chrystie Street.*

I have examined the National Arithmetic with much care, and I am very much pleased with it. I think the examples are appropriate, and I am so well satisfied that I have recommended the work to the board of Trustees of the 11th Ward School, and they have adopted it in use. I cheerfully recommend its adoption in all our schools.

SENECA DURAND,
Principal 11th Ward Common School of the City of N. York.
New York, Jan'y 25, 1844.

I have used Greenleaf's Arithmetic in my school since 1839, and it gives me pleasure to say that I know of no work so well adapted to the purposes of promoting the industry and securing the improvement of the pupils as this. The rules that precede the numerous examples are encumbered by no long and valueless explanations. Everything is plain and to the point. I think that every teacher who makes a trial of the above treatise will give it a decided preference over any work of the kind hitherto published, and I cordially join with many who regard it as the "*National Arithmetic.*"

E. H. JENNY, A. M.
Principal of Monroe St. School, and New York Institute, 230 East Broadway.

New York, June 1, 1844.

B. Greenleaf, Esq. Dear Sir: I have used the National Arithmetic and Introduction in the Eleventh Ward School, the last six months, and take pleasure in expressing to you my satisfaction in them, as superior to any books in this branch of education with which I am acquainted. I find them far superior to the treatise which I used before we adopted yours. The Introduction, I consider very valuable, as the expense renders it practicable to a large number of pupils, with the use of which *alone*, I can make good arithmeticians.

Very respectfully, S. DURAND,
New York, May 31, 1844. *Principal.*

Having for some time used Greenleaf's Arithmetic, it affords me pleasure to say that I fully concur with the above in giving it preference over any treatise on the subject, that has come under my notice. Respectfully,

WILLIAM S. HALL.

I have examined the "National Arithmetic," by Mr. B. Greenleaf, of which I entertain a very high opinion. It seems to me to be a work of great merit. The peculiar excellence of most of its rules and the multitude and variety of its examples, illustrating and exercising each, are such as to render the scholar's rapid advancement in this science, certain; while it, by these means, affords the greatest convenience to the teacher. But among the many excellences of this book, I would particularly mention the plan of introducing each rule by familiar examples in "Mental Arithmetic," so simple, as intuitively to be understood, and yet so adapted to each particular rule as fully to explain and impress it on the mind. This book is calculated not only to teach Arithmetic efficiently, but also simultaneously to initiate the pupil in the more important branch of education, *to think*.

The chapters on Geometry, Book-keeping, &c., seem well written and adapted to the subjects which they are intended to explain. I think should this work be introduced into our schools, in place of many inferior, and less efficient, great advantages would be experienced. HENRY KIDDLE,
New York, Nov. 20, 1843. *Teacher District School No.* 1.

I have examined "Greenleaf's National Arithmetic," and can with confidence recommend it as a work well adapted to the wants of teachers. The author has made it not "to suit the meanest capacity," but so that in the hands of a good Instructor it may help to expand the capacities of all, and I have no doubt that the book will in due time be extensively used.

HENRY L. OGDEN, *Teacher District School No.* 3.

I concur in the above.

THOS. M. GAHAGAN, *do. do. do. do.* 2.
New York, Nov. 29, 1843.

I have occasionally used Mr. B. Greenleaf's National Arithmetic as a reference for practical sums, in the senior classes of the school that I have charge of, for the past eighteen months; and most cordially congratulate the author that I find it *all* he has *promised*, and *all* that teachers and others have recommended it to be, both in arrangement and practicability. I believe it *one* of the *best*, if not *the best* work of the kind now in use, for advanced pupils in all schools, and with others, cheerfully recommend its perusal and use to teachers and superintendents of schools generally. JNO. W. KETCHAM,
New York, Oct. 4, 1843. *Principal New York Public School No.* 7.

I have used Greenleaf's National Arithmetic for some time in my school, and believe it to be the best work on that subject now in use.

D. PATTERSON,
Teacher Male Monitorial School, and of Public School No. 3.
New York, Oct. 11, 1843.

I have given an attentive perusal to "Greenleaf's Arithmetic," and I am clearly of opinion that we have no work in use better calculated for the higher classes in our common schools. The arrangement is good — the rules clear and concise — and the examples more numerous than in most of the books in ordinary use. The decided advantage of the latter quality, in our schools at least, is too obvious to teachers to need recommendation; its chief merit, however, being that, by constant practice, the rules are strongly impressed upon the minds of the pupils.

The system of Book-keeping, appended to the Arithmetic, recommends itself strongly to me, on account of its clearness and brevity; and from it, in my opinion, a pupil will obtain as good a practical knowledge of that acquirement, in a much shorter time, as he would by wading through the mysteries and intricacies of more voluminous works.

Upon the whole, I have no hesitation in saying that it is one of the best works of the kind that has come under my observation; and this conclusion is partly forced upon me by, to me, the best of all possible tests — that of having used it in my classes.

JAMES B. O'DONNELL,
Nov. 3, 1843. *District School, No.* 4, *New York.*

I cheerfully concur in the above recommendation. JOHN OAKLEY,
Principal of District School, No. 4, *New York City.*

I cheerfully subscribe to the above, having used it for several years in my own school.

E. H. JENNY, A. M.,
Principal of New York Institute.

We have examined the National Arithmetic, by B. Greenleaf, and think it advantageously compares with the best books on the subject in use. It is a work practical in its character, full and diversified in its examples, and we think well adapted to the wants of our country.

JOSEPH MCKEEN,
LEONARD HAZELTINE,
WM. BELDEN,
ABM. K. VAN VLECK,
A. V. STOUT,
DAVID PATTERSON,
J. W. KETCHAM,
Teachers of Public Schools.

New York, Nov. 29, 1843.

I have carefully examined Mr. Greenleaf's Introductory and National Arithmetics. I give them the preference to any work on that subject which I have hitherto seen.

J. PATTERSON,
New York, Nov. 29, 1843. *Teacher Pub. School No.* 4.

I fully concur in the above recommendations of Greenleaf's Arithmetics.

E. H. JENNY, A. M.,
Principal New York Institute.

I have examined, and introduced "Greenleaf's Introductory" and "National Arithmetic" into my school. It needs no recommendation. Its worth will be found upon every page.

J. PALMER, JR.
Grand Street Academy, New York, October 30, 1843.

Mr. B. Greenleaf: I have carefully examined your Arithmetic, and consider it the best work on the subject I am acquainted with. I have accordingly introduced it into the school under my charge.

Yours, ROBT. H. BROWNE,
Principal of the School attached to the Scotch Presbyterian Church, N. Y.

New York, Oct. 10, 1843.

We have examined Greenleaf's Arithmetic, and the result is, that we have allowed it immediately to supersede Emerson's, which we have had in use for some time past.

PERRINE & FOIGNET,
Classical French and English School, 739 Broadway.

B. Greenleaf, Esq. Dear Sir: I have carefully examined your National Arithmetic. The plan and execution of the work meet my entire approbation. It is evidently the production of an author who is not only a mathematician, but a practical instructor. Its rules are concise, definite, and clear; the explanations lucid and copious, without being prolix or redundant. I consider it, upon the whole, by far the best work in use.

No pupil can study it throughout, under an intelligent teacher, without becoming familiar with arithmetical science. I cheerfully recommend it as the best text book for schools.

JAMES S. EVANS, A. M.
New York, April 20, 1844.

Sir: I have examined the National Arithmetic, by B. Greenleaf, Esq., and consider it decidedly the *very best* with which I am acquainted. I have introduced it into both departments of my school.

Yours, &c.,
JOSHUA BUTTS,
Principal of the Young Ladies' and Gentlemen's Seminary, 112 King Street.
New York, Sept. 20, 1843.

After a careful examination of Greenleaf's Introduction and National Arithmetics, we have introduced them into both departments of our School. In our opinion it is decidedly superior to any other system with which we are acquainted. It need only be known, and we predict for it a rapid and extensive circulation.

WM. MOORE, } *Principals New England*
H. C. FISH, } *School, 327 Bowery.*
New York, Jan'y 1st, 1844.

Having thoroughly examined Greenleaf's Arithmetic, I feel myself perfectly justified in saying that in point of excellence of definition, fullness of illustration, and adaptedness to the necessities of those for whose especial benefit it has been prepared, it sustains a very high rank among similar publications. I have recommended it to those institutions with which I am professionally connected.

JACOB T. BERGEN,
New York, May 30th, 1844. *Teacher of Mathematics.*

Having used Greenleaf's National Arithmetic as a text book, I cheerfully recommend it to the favorable notice of those interested in the instruction of youth, as being unsurpassed by any treatise on Arithmetic with which I am acquainted, in simplicity of arrangement, clearness and perspicuity in the rules, and adaptation to use in schools. It is well calculated to facilitate the progress of the pupil, whether intending to pursue a full mathematical course, or, by practical illustrations, to be fitted for the counting house. J. R. TAYLOR,
New York, April 26, 1844. *Teacher of Mathematics.*

I have examined Greenleaf's National Arithmetic, with much pleasure The perspicuity of its arrangement, and the clearness and brevity of its rules; the numerous examples which it contains, and which are well adapted to the purposes of practical business, are its great recommendations. In fact, I consider it equal, if not superior, to any treatise on the subject we at present possess.

JAMES F. MACULLY,
New York, Dec. 9, 1843. *Teacher of Mathematics*

Within a few years, many Arithmetics of real merit have been published. It is a happy circumstance, that *competent, practical* men, are thus inclined to benefit the community by the results of their investigation and calculation in the essentially important science of numbers.

Among the Treatises, I know of none that more *happily combines; lucidly defines;* and *wisely arranges* what is *necessary* and *useful*, in a popular arithmetical compilation, than Mr. Greenleaf's National Arithmetic.

D. STEVENS,
Principal of the Select Academy, 172, *Spring St., N. Y.*

New York, Dec. 14, 1843.

I concur in the above views.

S. AUSTIN,
Principal Select School, 140, *Laurens St.*

We, the undersigned, have examined Greenleaf's National Arithmetic with considerable attention and interest, and have used it as a text book, for several months, with entire satisfaction. We regard it as among the best works on the science of which it treats.

G. A. ROGERS,
SAMUEL S. POTTER,
HENRY OSBORN,
} *St. Luke's School.*

New York, June 1st, 1844.

We have examined Greenleaf's National Arithmetic. We find it equal, and in many respects superior, to most Arithmetics now in use. There is clearness and method in the arrangement; a happy combination of the analytic and synthetic; and its principles are illustrated by examples so numerous, and in their form so modern and practical, that the pupil feels himself in the study of them actually at the store and the counting house, engaged in the business of life.

We doubt not the utility of this work will give it a circulation which will fully entitle it to be a Text Book for the Nation. We shall introduce it into our school.

JAMES FANNING,
R. H. CADY,
} *Principals Commercial and Collegiate Academy.*

New York, Dec. 25, 1843.

After an examination of Greenleaf's National Arithmetic, I think it the best treatise on the subject, without exception, that I have ever seen. The excellent arrangement of its parts, the clearness and brevity of its rules, and the number and variety of examples under each, are features that distinguish it among all works of the kind. The author, understanding the duty of the teacher, has rendered his work the more valuable by the omission, under each rule, of a mass of explanation useless to the pupil, which, if necessary, can be supplied with more effect by a competent instructor.

A. DICKINSON.

New York, Nov. 23, 1843.

I think Greenleaf's National Arithmetic the best with which I am acquainted, having introduced it into my school, and most cheerfully recommend its immediate introduction into all the schools of our beloved country.

M. BEARDSLEY,
Principal of the Female Seminary, Waverly Place, New York.

New York, May 29th, 1844.

On examination of Mr. Greenleaf's Arithmetic, I have been pleased with its arrangement, simplicity, comprehensiveness, and variety of examples, and shall therefore introduce it as one of the text books on that science, into my school.

J. F. WORTH.

New York, Sept. 29, 1843.

I have used "*Greenleaf's Arithmetic*" in my school nearly three years, as a text book for my more advanced pupils, with much satisfaction. His "Introduction" (an excellent work for beginners) has just been put into my hands, and the two in connection form a *system*, in my estimation, second to none that has come under my observation.

JAMES LAWSON,
New York, Sept. 12, 1843. *Classical and English Teacher*, 101 *Grand St.*

"Greenleaf's Introductory Arithmetic," and his larger work, styled the "National Arithmetic, are works of the highest merit; and, in connexion, form the most *complete system on numbers* with which I am acquainted. I have used them in my school for several years, and have never seen any cause for regretting their adoption.

JAMES LAWSON,
New York, Jan. 27, 1844. *Classical and English School*, 101 *Grand St.*

New York, Oct. 3, 1843.

Sir: Having examined with much care your National Arithmetic, I consider it decidedly the best school book of the kind with which I am acquainted. The work needs only a careful examination by teachers to secure its immediate and extensive introduction.

C. BLOODGOOD,
B. GREENLEAF, Esq. *Principal Sixth Street Academy.*

I have examined "Greenleaf's Introduction" to the "National Arithmetic," and consider it, in general, well adapted to the purpose for which it was designed. I have also used the "National Arithmetic," itself, with a good degree of satisfaction. I can cheerfully recommend it as a good work for higher classes.

AARON RAND,
New York, Oct. 30, 1843. *English and Classical School*, 374 *Pearl St.*

Upon a perusal of the "National Arithmetic," of which Mr. Greenleaf is the author, I beg to give my hearty concurrence to the numerous testimonials already given; considering it a work of sterling merit, and decidedly superior to any other with which I am acquainted, I shall introduce it in my school forthwith.

SAMPSON M. SMITH,
New York, October 30, 1843. *Carmine Street Academy.*

After having examined Greenleaf's Arithmetic, with some care, I am prepared to say that it combines many excellences not found in most works on the same subject. I regard its arrangements as very judicious, and its illustrations as clear and practical. The sections on Geometry and Book-keeping will be found very important additions. I shall introduce the work into my school as soon as practicable, and have full confidence that it will be approved by all who make themselves thoroughly acquainted with it.

P. E. FARNSWORTH,
Principal of King Street Seminary, New York.

I have examined Greenleaf's Introductory, and National Arithmetics, and am much pleased with their completeness and systematic arrangement. Perhaps the best proof which I can give of the estimation in which I hold them, is, to say that I have introduced them into my school.

B. F. PARSONS,
Principal of Female Classical School.

Brooklyn, N. Y., Nov. 2, 1843.

Having examined "Greenleaf's National Arithmetic," I am prepared unhesitatingly to give it the preference to any Arithmetic with which I am acquainted.

A careful perusal of it is only necessary to convince any one of its superior merits. I shall use my influence for its introduction.

W. S. SPAULDING,
Principal of Brooklyn Female Seminary.

Having given considerable attention to the National Arithmetic and Introduction, which you left with me, I am led to prefer them to any other system of the science with which I am acquainted. The mental operations in several of the rules recommend the work, as they combine Arithmetic *with* and *without* the slate, and secure the advantages of both. The author has restored the doctrine of Ratio to its true definition, and it is hoped his view of the subject will prevail, and correct the confusion which some writers have produced in this part of Mathematical science. So far as I have examined and used these volumes, they appear well suited to aid youth in this branch of their education, and the author merits the reward of well directed efforts to facilitate the labors both of the instructor and the pupil.

NOAH H. WELLS,
Newark, July 17, 1844. *Teacher of Select School.*

I have, for several months past, used "Greenleaf's Introductory Arithmetic." It seems well suited to the business of the School Room, and is evidently the production of a practical teacher. Its arrangement is lucid and orderly; its exercises are judiciously prepared, and its rules occupy but a small space. It is not only a suitable *introduction* to a larger work, but is sufficiently copious for a large majority of those pupils who study arithmetic.

NATHAN HEDGES.
Newark, July 10, 1844.

Having used Greenleaf's Introduction, I feel willing to concur in the above recommendation.

H. F. WILCOX.

Having also used the above-named Arithmetic, I feel no hesitancy in agreeing with the above.

J. H. WALTON.
Newark, July 10, 1844.

I have examined Greenleaf's works on Arithmetic, with a considerable degree of attention. I admire their arrangement, and the clear and lucid manner in which their instructions are given. But I prefer them to other works of the kind which have fallen under my notice, chiefly on account of the great number and variety of the examples they contain. I should be pleased to see them introduced into all the public schools of this town.

J. J. CLUTE,
June 25, 1844. *Supt. of Common Schools of Castleton, Richmond Co.*

New York, June 13, 1844.

To B. GREENLEAF. I have, with pleasure, examined thy Arithmetics, the National and Introductory, and think them superior to any others with which I am acquainted. I am especially pleased with the accuracy and precision of the definitions and rules, also with the numerous examples well calculated to lead the pupil by easy and progressive steps, through the difficulties of the science. Perhaps the best proof of the estimation in which I hold their merits, is the fact that I have introduced them into the school under my care.

Thine,
SAMUEL BROWN,
Principal of Friends Monthly Meeting School, New York.

New York, July 25, 1844.

TO THE PUBLISHER: Dear Sir,—After using Greenleaf's Arithmetic as our only text book, long enough to ascertain its contents and character, I feel no hesitation in saying it is (for us, at least,) the best work extant. The author deserves much credit for the *clear* yet *concise* style in which the rules are communicated; also, the familiar and attractive expression of his questions.

Yours, most Respectfully,
NELSON MOWRY,
112 *King Street.*

I have not only examined Greenleaf's National Arithmetic faithfully, but I have tested its value by use; and I do unhesitatingly pronounce it the best Text Book of the kind, that has fallen under my notice, during my entire school-room experience. I shall, hereafter, use no other.

JOSEPH MCKEE,
Classical and English Teacher, Madame Chegaray's School.
Union Square, N. Y., July, 1844.

An attentive examination of Greenleaf's National Arithmetic, and the Introduction thereto, has confirmed the favorable opinion I had previously formed from report. The rules, simply yet accurately expressed, the lucid illustrations and copious examples, in fact, the entire arrangement show them to be works of no ordinary merit, and indicate that they have been prepared by one who is a complete master of the science. I have not the slightest hesitation in giving them a decided preference to any works of the kind I have hitherto seen.

M. J. O'DONNELL,
New York, Nov. 22, 1844. *Principal of Public School No.* 11.

I have examined Greenleaf's Arithmetic, and consider it among the best treatises on that subject, and as preferable to any other in use. I have adopted it as the text book of this Institution.

GEO. L. LEROW,
August 16, 1844. *Principal of New Brighton Collegiate School.*

I have, with care and much satisfaction, examined Mr. Greenleaf's Introductory and National Arithmetics, and think they are admirably adapted to the use of Common Schools and Academies. I have long felt the need of something different from what we had in Schools generally, to give the student a practical knowledge of Arithmetic, without multiplicity of questions, so arranged as to confuse the mind rather than unfold to it the principles of the science. The present works are calculated to meet the exigencies of the case, and I most cheerfully recommend them to others of my profession, and to the students under my tuition.

L. J. BRIDGMAN,
Principal of Port Richmond Academy.
Port Richmond, Staten Island, July 8, 1844.

Dear Sir:—I have examined, with great satisfaction, "Greenleaf's Arithmetic," and it gives me pleasure to say, that I consider it among the best of our elementary treatises. I shall embrace the earliest opportunity of introducing it into my School, and I hope the attention of all will be turned to a book promising so much advantage to the rising generation.

Respectfully, Yours,
IRA K. BALL,
Port Richmond, July 18, 1844. *Teacher of District No.* 6, *Northfield.*

Northfield, Staten Island, July 22, 1844.

Dear Sir:—A careful examination of Greenleaf's Introduction to the National Arithmetic, has convinced me of its superior excellence over any other (now in use,) with which I have been acquainted. Several circumstances enhance very much the value of this work. It contains nearly three times the usual quantity of matter in works of this class, and is rendered the cheapest book in this department of science, I have ever seen. The necessity of a work of this description will scarcely be questioned by any who have had experience in teaching Arithmetic. It seems to comprehend the whole science of numbers (so far as is necessary for any common branch of business) in the least possible space. It contains more useful matter, is better arranged for school use, and its rules are more clearly demonstrated, than any other work of the kind I have ever used. In one word, it seems to be the very book which every judicious teacher should select for his pupils.

E. GATES,
Formerly Assistant Teacher of Teacher's Seminary, East Hartland, Conn.

GREENLEAF'S NATIONAL ARITHMETIC.

Portsmouth, Aug. 5, 1838.

Benjamin Greenleaf, Esq. Dear Sir: Having examined, and, to some extent, introduced into our Schools the National Arithmetic, of which you are the author, we deem it a duty we owe to the public, no less than to yourself, to express our decided approbation of its merits. The method, arrangement, and quantum of matter it contains, the clear and lucid manner in which its rules are demonstrated, together with its adaptation to the wants of the community, entitle it, in our humble belief, to the patronage of every lover of scientific investigation.

Signed,

HAZEN PICKERING, JOHN T. TASKER,
A. M. HOYT, JOHN J. LANE,
JAMES HOYT, EDWARD J. LAIGHTON.
C. E. POTTER,

School Teachers of Portsmouth, N. H.

From Rev. Dr. Hopkins, President of William's College.

My opinion of Greenleaf's Arithmetic is, that it is adapted to give a more thorough knowledge of that science, than any other that I have seen.

Respectfully, yours, M. HOPKINS.

Williamstown, Dec. 30, 1837.

Poughkeepsie Institute, Jan. 9, 1839.

We have carefully examined the National Arithmetic, and do not hesitate in pronouncing it the best work of the kind which has come under our notice. The deduction of the rule from the operations is, in our opinion, the proper method; and the copious examples, under the various rules, are well selected and arranged. We hope it may meet with its merited success. We shall endeavour to extend and establish its use.

Yours, respectfully,
J. L. DUSINBERY, } *Principals.*
A. H. TOBEY, }

I have examined Greenleaf's Arithmetic, and consider it, in many respects, preferable to any work of the kind with which I am acquainted. I am particularly pleased with his illustration of the Square and Cube Roots, and the Rule of Proportion, and with the introduction of practical instruction on the subject of Banking, Custom-House Duties, Assessment of Taxes, &c. I think its introduction into Schools and Academies will prove of general interest to all who wish to acquire a knowledge of Arithmetic.

A. B. BULLOCK.

Hudson, Dec. 7, 1838.

I fully concur in the above, and shall use my influence to introduce it into my school. C. GREENE.

From the Principal of the Dutchess County Academy.

After a careful and comparative examination of Greenleaf's Arithmetic, I unhesitatingly say, I think it superior to any other Arithmetic within my knowledge. I shall with pleasure use my influence to give it a circulation in the Schools of this vicinity. WM. JENNOY

Poughkeepsie, Jan. 9, 1839.

I fully accord with Mr Jennoy in his opinion of Mr. Greenleaf's Arithmetic, and shall esteem it a privilege to recommend its use whenever an opportunity presents. O. M. SMITH,

Newburgh, January, 1839. *Principal of Newburgh High School.*

GREENLEAF'S NATIONAL ARITHMETIC.

I have examined, with considerable care and entire satisfaction, the System of Arithmetic by B. Greenleaf. I can say, without hesitation, I think it the most complete and well-arranged School System, in this branch of science, extant, and better calculated than any other to prepare our youth for active usefulness in all those pursuits where a knowledge of Arithmetic is requisite. I might speak of the happy combination of the Analytic and Synthetic methods of operation, and the still happier union of clearness with brevity in all the Rules and Definitions; but all this will be seen and pleasingly felt by those who peruse or study this truly valuable book. I shall do what I may, in my limited sphere of influence, to promote its introduction into the Schools of our State.

Albany, Dec. 1838. S. STEELE, *Teacher.*

I have examined Greenleaf's National Arithmetic, and am of opinion, from its practical character and the order of the arrangement, that it is well calculated to induct the inquiring pupil into the useful business operations of the community, for which the study of Arithmetic is designed. I shall not hesitate to recommend it to my own pupils and to the teachers of other Schools.

EDWARD SMALL,

Albany, Dec. 1, 1838. *Teacher of the Lancaster School, Albany.*

Mr. Greenleaf. Sir: I have examined your National Arithmetic and am glad to say, it meets my approbation; and I think I shall introduce it into my School, to the exclusion of all others. A. P. SMITH,

Albany, Nov. 28, 1838. *Teacher of the Second Public School, Albany.*

Mr. Greenleaf. Dear Sir: I have examined your System of Arithmetic, and am happy to state, that it meets with my unqualified approbation, and that I shall immediately introduce it into my School. Yours, respectfully,

Albany, Nov. 27, 1838. THOMAS McKEE.

We fully concur in the above.

NEWMAN & WALLACE, *Teachers, Mechanics Academy, Albany.*
D. E. BASSETT, *Principal of an Academy,* *Do.*
JOEL MARBLE, *Principal of District School, State Street,* *Do.*
J. W. BULKLEY, *Principal of an Academy,* *Do.*

From Dr. Fox, Principal of the Boylston School, Boston.

B. Greenleaf, Esq. Dear Sir: I have just been examining your new Arithmetic, and think it an excellent work. I like the plan of it much. Among its many excellences I perceive the following, viz. — The Tables of Money, Weights, and Measures carried out to the lowest denomination; the great variety of examples under each Rule, and likewise your method of treating several parts of the science, as Fractions, Proportion, Evolution, and Exchange, — every thing concerning them must appear clear, I think, to the student. The Geometry, Philosophical Problems, Mechanical Powers, and Book-keeping, seem also to be handled in a perspicuous manner. The Rules of Cross Multiplication and Position, I am happy to see have place in the work; for, after all, they are too useful, the latter especially, to be omitted in our arithmetical treatises. On the whole, the work appears to me well calculated to lead youth to a clear and thorough knowledge of the various branches of this Science, and I doubt not it will be sought after, as an improvement on former works of the kind, and obtain an extensive circulation. Yours, respectfully, CHARLES FOX.

A thorough examination of Mr. Greenleaf's Arithmetic has induced me to introduce it into the Academy with which I am connected. The arrangement is excellent, and much valuable matter is found in the National Arithmetic, not contained in others now in use. Very respectfully, yours,

Barnstable, Dec. 9, 1837. F. A. CHOATE

From Mr. J. P. Engles, A. M., Principal of the Classical Institute, Philadelphia.

I have examined, with considerable interest, Greenleaf's National Arithmetic, and have no hesitation in recommending it as an admirable system of Arithmetic, which contains all that is essential to a knowledge of the science, and nothing that is useless. The arrangement, too, is such as to make the contents easily available to the teacher and the pupil. Should it succeed in displacing the host of so called "Assistants," with which our schools are flooded, I conceive it would be equally to the comfort of teachers, and the profit of students. I shall cheerfully introduce it into my Academy. J. P. ENGLES.

Philadelphia, Nov. 14, 1838.

I cheerfully concur in sentiment with Mr. Engles, respecting Mr. Greenleaf's Arithmetic; it is the best work of the kind I have ever seen. With a great deal of pleasure, I shall introduce the same into my Seminary.

W. ALEXANDER, *Classical Teacher, Philadelphia.*

I have examined Greenleaf's National Arithmetic with a great deal of satisfaction, and have no hesitation in saying, that it is the most complete system of Mercantile Arithmetic with which I am acquainted; and will cheerfully recommend it as occasion may require.

E. GRIFFITHS, *Teacher of Mathematics, Philadelphia.*

Philadelphia, Nov. 12, 1838.

The undersigned entirely concur in the opinions expressed by Messrs. Engles, Alexander, and Griffiths, respecting Mr. Greenleaf's Arithmetic.

JOHN W. FAIRES,
B. P. HUNT,
JAMES P. ESPY,
} *Teachers in Philadelphia.*

I have examined Mr. Greenleaf's National Arithmetic with some care, and am much pleased with its arrangement; his examples, under each rule, are numerous and appropriate: I am so well satisfied, that I intend to introduce it into my Seminary. THOMAS MCADAM.

Philadelphia, Nov. 14, 1838.

We fully concur with the gentlemen, who have already given recommendations of the National Arithmetic, considering the work well calculated to give youth a correct knowledge of the principles of Arithmetic.

WM. VOGDES,
E. O. KENDALL,
} *Philadelphia Centre High School.*

Copy of a letter from G. W. Harby, Esq., Principal of Harby's Academy, New Orleans, addressed to the Publishers.

Gentlemen: Viewing the publication of School Books of the first importance, it was with much pleasure that I received Greenleaf's National Arithmetic. For fifteen years, and upwards, I have devoted my life to the instruction of youth, during which time many Arithmetics have fallen under my inspection. I take a strong interest in every work that pertains to mathematical learning, and unhesitatingly pronounce Greenleaf's Arithmetic an important treasure to Academies; it is fraught with a great deal of care, and in an easy, plain, and uniform style. His Geometrical, Mechanical, and Astronomical Problems are concise and clear: they lead the youthful mind to the exercise of a little patience,—not so arduous as to fatigue, but sufficiently laborious to call the mental faculties into exercise, and to create a taste for mathematical knowledge, and for scientific discovery and invention,—which has lately so conspicuously crowned some of our countrymen with brilliant success. I shall make it the standard book in my Institution, and recommend it to others of my profession.

I remain, gentlemen, your obedient servant,

New Orleans, August 22, 1838. GEORGE W. HARBY.

GREENLEAF'S NATIONAL ARITHMETIC.

I have recently examined Greenleaf's National Arithmetic, and am well pleased with the work. It contains much important information in reference to mercantile pursuits, and, in my judgment, is well adapted to the wants of our increasing number of Schools and Academies. I regard the systems of Book-Keeping, contained in it, as very important.

EDWARD WILSON, Jr.
Troy, N. Y. Nov. 28, 1838. *Teacher, Troy Monitorial School.*

I have examined Mr. Greenleaf's National Arithmetic, and do not hesitate to say, that it is not only a practical and valuable work, but an admirable one; one that is every way calculated to produce an interest in the student, and to facilitate his advancement in the science of numbers. I shall use my influence in introducing it into my School.

JOSEPH CHILDS, Jr.
Troy, Nov. 1838. *Teacher, 5th St. School, Troy.*

I have examined Greenleaf's National Arithmetic, and cheerfully concur in the above recommendations; and shall use my influence in introducing it into my School.

JAMES PARK,
Troy, Nov. 28, 1838. *Teacher, 4th St. Academy.*

Poughkeepsie, (N. Y.) Jan. 1, 1839.

After a cursory examination of Greenleaf's Arithmetic, I have no hesitation in awarding to it a large amount of arithmetical knowledge, more, indeed, than almost any work within my acquaintance. The youth, who should go through this work carefully and thoroughly, could not fail of obtaining a familiar acquaintance with the properties and powers, and various applications, of numbers.

A. LATHROP,
Teacher of Mathematics in the Poughkeepsie Collegiate School.

The following is the *conclusion* of a critical notice of the work, from Thomas M. Brewer, Esq., Principal of an Academy, Poughkeepsie.

Upon the whole, considering the judicious arrangement, the adaptation of the Examples to the business requirements of our country, the perspicuity of illustration, and the extensive range of Arithmetical Science embraced, I am happy to say, I think the book worthy of its title. And I shall take an early opportunity to introduce it into my Institution as the text-book best adapted to our use of any I have seen.

THOMAS M. BREWER.
Poughkeepsie, Dec. 14, 1838.

I have examined Greenleaf's National Arithmetic with much care, and hesitate not to pronounce it a very valuable work, superior in many respects to any Arithmetic, now in use, with which I am acquainted. I shall immediately introduce it into my School, and most cheerfully recommend it to the public, believing it well adapted to the wants of our Schools and Academies.

Yours, very respectfully,
JAMES H. HOWE,
Principal of the Lancasterian School, Poughkeepsie.
Poughkeepsie, Dec. 17, 1838.

Benjamin Greenleaf, Esq. Sir: I have had the pleasure of examining your Arithmetic, and, among the many I have used in teaching, have never found one without some deficiency until yours came into my hands. I think it preferable to any other in its arrangement, the lucid illustration of its principles, and the great amount of matter it contains. I shall feel in duty bound to introduce and recommend it when an opportunity offers.

Yours, with respect,
A. KIDDER,
Poughkeepsie, Jan. 1, 1839. *Teacher of a Select School.*

GREENLEAF'S ARITHMETICS are used in the following Schools and Academies, among others, in New York City, which are of the highest respectability.

Washington Institute,	T. D. & T. W. Porter,	Prin'als.
Rutger's Female Institute,	Charles E. West.	Principal.
Cornelius Institute,	Rev. J. J. Owen,	"
All Saints' Parochial School,	Wm. A. Taylor	"
Commercial and Collegiate School,	Hubbs & Clarke	"
Classical, Mathematical, and Com'l Institution,	H. Peugnet	"
Classical and English School,	W. King	"
Classical and English School,	Chas. W. Feeks	"
New York Institute,	E. H. Jenny	"
Trinity School,	William Morris	"
Friends' Monthly Meeting School,	Samuel Brown	"
" *Female Monthly Meeting School*,	Misses Smith & Carpenter	"
Boarding and Day School for Young Ladies,	Madame Chegaray	"
Mechanics' Institute School,	John M. Reid	"
Select School for Boys,	Daniel P. Bacon	"
English and Classical School,	Aaron Rand	"
Classical, Mathematical, and English Academy,	J. F. Worth	"
Classical and English School	C D. Jackson	"
Select School for Boys,	J. J. Greenough	"
Classical, French, and English School,	Perrine & Foignet	"
New England School,	Moore & Fish	"
Scotch Presbyterian School,	Robert H. Browne	"
City Commercial School,	J. B. Quick	"
Classical and English School,	Joshua Butts	"
Academy for Young Gentlemen,	E. L. Avery	"
Commercial and Collegiate Institute,	J. Fanning & H. Cady	"
English, Classical, and Mathematical School,	A. Dickinson	"
English and Classical School,	M. Frey	"
Select School,	B. Fowler	"
St. Luke's School,	G. A. Rogers, S. S. Potter, & H. Osborn	"
Female Academy,	Mrs. Page	"
Classical and English School,	James Lawson	"
Young Ladies' School,	Mrs. S. C. Reed & Daughter	"
Select Classical and English School,	R. J. L. Austin	"
Seminary for Young Ladies and Gentlemen,	Nelson Mowry	"
Grand Street Academy,	S. Palmer, Jr.	"
Carmine Street Academy,	S. M. Smith	"
Sixth Street Academy,	C. Bloodgood	"
English and Classical School,	Mess. C. Tracy & Drew	"
Select School for Young Ladies, (Brooklyn,)	A. Greenleaf	"
Female Seminary, (Brooklyn,)	W. S. Spaulding	"
Female Classical School, (Brooklyn,)	B. F. Parsons	"
Collegiate and Commercial School, (Brooklyn,)	James G. Russell	"
Brooklyn Grammar School. "	Walter Chisholm	"

This system of Arithmetic is also used in the "*Normal Schools*," and the *District Schools* in the City, and in various parts of the State of New York, and has recently been adopted in the Public Schools of the city of Newark, N. J.

These Arithmetics are the text books in a large number of the best Schools. and Academies in New England, among which are the following:

Normal School	at Bridgewater, (Mass.,)	N. Tillinghast, Esq.	Principal
" "	at Lexington, "	Rev. Samuel J. May	"
Phillips Academy	at Andover, "	Rev. Sam'l H. Taylor	"
" "	at Exeter, (N. H.,)	Mr. Gideon Soule	"
Young Ladies' Inst.	at Pittsfield, (Mass.,)	Rev. W. H. Tyler	"
Female Seminary	at Bradford, "	Miss A. C. Hasseltine	"

besides other distinguished institutions, and have been extensively adopted for public and private schools in various sections of the United States.